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Preferred dimension ratios of small rectangular rooms

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Abstract: Rooms for music rehearsal, sound studios, control rooms, etc., need a smooth frequency response. For that reason, the frequencies of the room modes should be spread as well as possible, and this is controlled by the aspect ratios of the dimensions. The relative variance of the frequency spreading of the lowest 25 room modes is applied as a quality criterion. The results have revealed that the length-width ratio is much more important than the width-height ratio. The length-width ratio should be within 1.15–1.45. The height can be chosen more freely without compromising the acoustical quality.

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1. Introduction

Rooms used for music, speech. or acoustical measurements need to have good acoustical properties. In small rooms, say up to 300 m^3, it is a challenge that the density of room modes is sparse at low frequencies. For that reason, it is important to design the rooms in such a way that the room modes at low frequencies are spread as well as possible. The low frequency range is here defined as 20–200 Hz. The frequency distribution of the room modes is dictated by the room dimensions or, more precisely, by the aspect ratio between the dimensions.

The upper limit of volume where the analysis is valid is not exact. It can be related to the physics, i.e., the dimensions related to the wavelength, or to the human perception of sound. Setting the lower limit of the frequency range to 20 Hz, the lowest mode will be below that frequency if the longest dimension exceeds 8.6 m. With the most common aspect ratios of the room, this longest dimension corresponds quite well with a volume around 300 m^3.

Talking, singing, or playing a musical instrument, the person acts as a sound source and listener at the same time. When the distance to one of the walls exceeds 8.6 m, the sound reflection from that wall arrives with a delay of more than 50 ms, which means that the reflection may be audible as an echo. Thus, it may be argued that the perception of the sound changes from the frequency domain to the time domain when the volume exceeds a limit around 300 m^3.

The study of acoustic quality of small rectangular rooms and the importance of aspect ratios has been the topic of a vast number of research papers since the early days of room acoustics around 1900. The results have often been in the form of suggested specific optimum aspect ratios. A rectangular room is characterized by the dimensions length (l), width (w), and height (h). In the following, it is assumed that $l \geq w \geq h$. Consequently, if the height of a room happens to be greater than the width, the meaning of the symbols h and w is switched.

Traditionally, the aspect ratios have been presented by numbers normalized by the height, i.e., in the form (1:w/h:l/h). However, it is realized in the present study that a better representation is by the pair of ratios w/h and l/w, i.e., the ratio between the two smaller dimensions and the ratio between the two larger dimensions. It is found that these ratios do not have equal importance for the distribution of the normal modes and, consequently, for the acoustic quality of the room. While l/w should be within a narrow range, w/h has minor importance.

2. Previous work

Already Sabine (1900) commented on the question of room dimension ratios: "Thus the most definite and often repeated statements are such as the following, that the dimensions of a room should be in the ratio (2:3:5), or according to some writers, (1:1:2), and others, (2:3:4); it is probable that the basis of these suggestions is the ratio of harmonic intervals in music, but the connection is untraced and remote." Sabine was very skeptical of such suggestions, and with good reason; the mentioned dimension ratios are not good, and indeed (1:1:2) is the worst possible, as we shall see later.

Volkman (1942) suggested different ratios based on $2^{1/3}$ and $2^{2/3}$, and he presented a diagram with recommended ratios for different room sizes, e.g., (1:1.26:1.59) or rounded (1:1.25:1.6) for small rooms and rounded (1:1.6:2.5) for average sized rooms. The former ratio has often been used for the design of reverberation chambers for acoustical measurements.

A theoretically based study of aspect ratios in rectangular rooms was published by Bolt (1946). He studied the statistical distribution of the frequency interval between the modes in the lower frequency range. Approximately 25 modes were used within a lower and upper limiting frequency, depending on the volume. Thus, the results were to some extent dependent on the volume. The optimum aspect ratio was found to be around (3:4:5), i.e., (1:1.33:1.67).

Louden (1971) looked at the standard deviation of the frequency spacing of the modes. In the calculations, he used all modes up to a limiting frequency that was volume-dependent and not higher than six times the lowest mode. The best result was achieved with the ratio (1:1.4:1.9), while the worst was (1:1.4:2.8).

Walker (1993) wrote a report for the BBC on optimum aspect ratios for studios, control rooms, and listening rooms. He applied a mean square room quality index very similar to that of Bolt but based on all modes up to 120 Hz. Thus, the results would change a little with changing volume. For a 200 m^3 room, two optimum aspect ratios were identified, (1:1.19:1.40) for a high room and (1:1.75:2.2) for a room with more practical height, namely 3.7 instead of 4.9 m. Based on the analysis, Walker proposed a criterion for acceptable room proportions; $1.1 \cdot w/h \leq l/h \leq 4.5 \cdot w/h - 4$.

Cox and D'Antonio (2001) applied an image source model with source in one corner and receiver in the opposite corner to calculate the frequency response. By numerical optimization, the room dimensions were changed to achieve the flattest possible frequency response in the frequency range 20–200 Hz. They found the worst-case ratio to be (1:1.075:1.868), but they did not report any of the optimized aspect ratios.

A similar principle was applied recently by Meissner (2018), who looked at the calculated frequency response between 20 and 200 Hz. The smoothness of the frequency response was used as a criterion, comparing the frequency response with a second-order polynomial and the normalized correlation coefficient as criterion for the smoothness. A drawback that makes this method complicated is that the result depends not only on the room aspect ratio, but also on the absolute volume of the room and on the absorption coefficients of surfaces. Higher absorption means more shallow room modes with increased bandwidth and thus improved smoothness. A small volume means more focus on the lowest room modes, which naturally are more separated than the higher room modes. In a large room volume, the lowest room modes are below 20 Hz and thus not within the frequency range being analyzed. For a volume of 150 m^3, the following dimension ratios were found to produce very smooth frequency responses: A (1:1.20:1.45), B (1:1.40:1.89), and C (1:1.48:2.12). The letters A, B, and C are used as labels for these optima in the following. For a small volume of 50 m^3, two optima were detected, one close to C and another at (1:2.55:3.44). The problem with the latter is that the height of the room is only 1.8 m, which means that the result is not applicable in practice.

3. Normal modes in a rectangular room

A rectangular room with dimensions l, w, and h has normal modes with the natural frequencies f_i calculated with the formula

$$f_i = \frac{c}{2}\sqrt{\left(\frac{n_x}{l}\right)^2 + \left(\frac{n_y}{w}\right)^2 + \left(\frac{n_z}{h}\right)^2}. \tag{1}$$

Here, c is the speed of sound in air, and (n_x, n_y, n_z) are the modal numbers. The index i is the number of the mode when all room modes are arranged in order of increasing frequency. Thus, f_1 is the natural frequency of the first mode (1, 0, 0) when $l \geq w \geq h$.

The normal modes have a strong influence on the frequency response at low frequencies. For the frequency range 20–200 Hz, the frequency response is calculated for two rooms with the volume 150 m^3 and very different aspect ratios (see Fig. 1). All surfaces are assumed to have the absorption coefficient 0.2. The details of the calculation method are given by Rindel (2015, 2016).

The curves in Fig. 1 have peaks that can be related to the natural frequencies of the normal modes, and the peaks appear reasonably separated up to around 100 Hz. At higher frequencies, the modes are too close together to allow the individual modes to be identified. Comparing the two curves in Fig. 1 makes it clear that the cubical room has fewer and stronger peaks than the room with optimum aspect ratios. The reason is that many modes coincide at the same frequency in the cubical room. The frequency distances between the first 25 modes are displayed in Fig. 2.

While none of the modes coincide in the room with optimum aspect ratio, many modes have zero frequency interval in the cubical room, and only seven of the 24 intervals are non-zero. The frequency spacing of the modes can be used to quantify the acoustical properties of a room at low frequencies.

4. Coupling between a musical instrument and a room

Playing a musical instrument in a room means a coupling acoustically between the instrument and the room. Thus, the room behaves as an acoustic extension to the instrument. While a good room cannot compensate for a mediocre instrument, it may be assumed that an acoustically bad room can ruin the sound qualities of a good musical instrument. A good music room should support the musical sounds as well as possible, but without favoring some tones at the expense of other tones. Imagine playing on a piano a chromatic scale from C_1 (32.7 Hz) to C_2 (65.4 Hz) in each of the two rooms referred to in Fig. 1. In the cubical room, the sound level will vary up and down and the tones will sound unequal, while the variation in sound level throughout the scale will be very small when playing in the room with optimum dimension ratios.

While the importance of a smooth transfer function is undisputed in sound studios and control rooms, it can only be assumed to be equally important in music rehearsal and practice rooms. No evidence for this has been found in

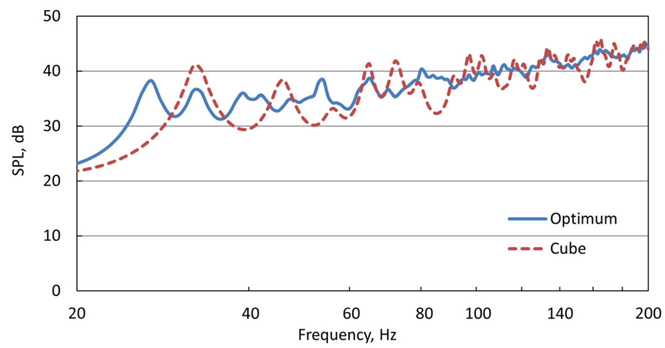

Fig. 1. Calculated frequency response between 20 and 200 Hz for two 150 m^3 rooms, one with optimum dimension ratios (1:1.20:1.45) and the other one with cubical shape (1:1:1). SPL, sound pressure level.

the acoustical literature. So, it is not known to what extent a musician can perceive and appreciate the difference between a room with good or bad distribution of the normal modes. This would be an interesting topic for future research.

5. Frequency spacing index (FSI)

The acoustical quality criterion applied here is based on the frequency distribution of the room modes. Thus, the volume and absorption properties are not involved. Whether this is an advantage or not can be discussed. On one side it is a fact that the smoothness of the frequency response increases with increasing absorption of the surfaces. On the other hand, increasing the absorption coefficient would tend to blur the effect of different aspect ratios, and with high absorption, the room approaches an anechoic room, which has a perfectly flat frequency response.

The FSI is the normalized relative variance of the intervals between the low frequency room modes when arranged in order of increasing frequency. The FSI $\psi(n)$ is calculated by the formula

$$\psi(n) = \frac{1}{n-1} \sum_{1}^{n-1} \left(\frac{\delta}{\bar{\delta}} \right)^2, \qquad (2)$$

where$n$ is the number of modes considered, f_1 is the frequency of the first mode, f_n is the frequency of mode number n, δ is the frequency difference between two neighboring modes, and the average frequency spacing is $\bar{\delta} = (f_n - f_1)/(n-1)$.

The FSI for the first 25 room modes $\psi(25)$ is applied here as a quantitative measure of the room acoustic quality at low frequencies. The FSI was first used by Bolt (1946) and later by Walker (1993), but in slightly different ways, namely within a lower and upper limiting frequency depending, on the volume.

The FSI should be as low as possible, and the (unrealistic) theoretical ideal is $\psi = 1$, corresponding to perfectly equal spacing of the modes. In a real room, the lowest possible FSI is $\psi = 1.3$ obtained for the aspect ratio (1:1.20:1.45). This is the same as the optimum A found by Meissner (2018) using a different criterion, namely the smoothness of the frequency response.

6. Results

The FSI is calculated as function of the two ratios w/h and l/w in steps of 0.05 within a representative range from 1 to 3 and from 1 to 2, respectively. The result is displayed as a contour-plot in Fig. 3.

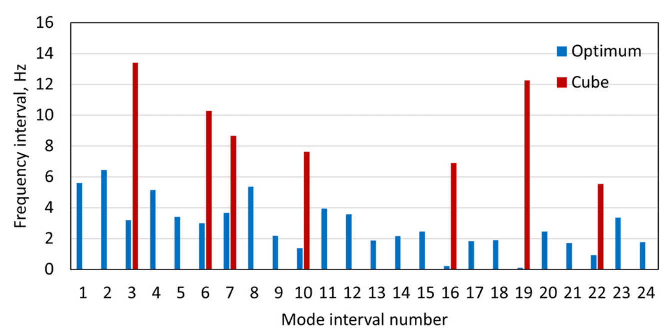

Fig. 2. The frequency intervals between the first 25 normal modes in two 150 m^3 rooms, one with optimum dimension ratios (1:1.20:1.45) and the other one with cubical shape.

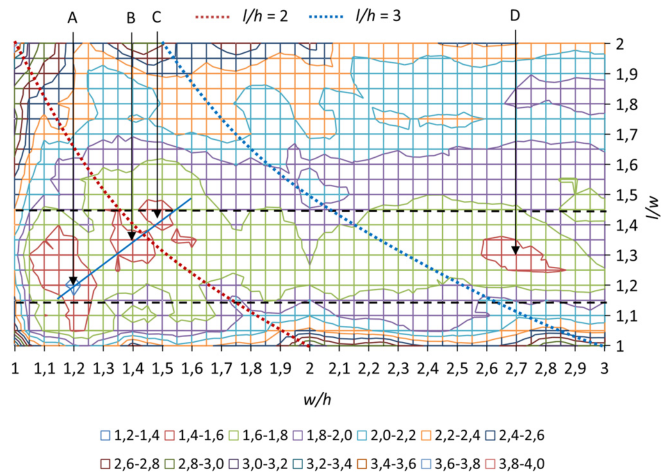

Fig. 3. Contour-plot of FSI $\psi(25)$ as a function of aspect ratios w/h and l/w. Inside the red regions where $\psi(25) \leq 1.6$, the spread of the modes is good. Outside the green region where $\psi(25) > 1.8$, the spread of the modes is poor, and the corresponding aspect ratios should be avoided. The solid blue line is the regression line for the three aspect ratios labelled A, B, and C. The two horizontal dashed lines mark the upper and lower limit of the proposed simple design criterion for l/w. The dashed red and blue curves indicate the l/h ratios 2 and 3, respectively.

The nearly optimum aspect ratios are those with $\psi(25) \leq 1.6$ shown in the red contour in Fig. 3. The centers of the red regions correspond very well with the optima A, B, and C found by Meissner (2018). An additional good region (D) is seen around $w/h = 2.7$ and $l/w = 1.3$.

Looking at Fig. 3, the most striking observation is the week dependency of w/h and the strong dependency of l/w. This means that the aspect ratio of the two larger dimensions is more important than the aspect ratio of the two smaller dimensions. The influence of w/h is so weak that the condition $w/h = 2$ only gives rise to a minor decrease in acoustic quality. The same is true for the condition $l/h = 2$. Thus, the traditional rule-of-thumb that the aspect ratio of 2 between any of the room dimensions should be avoided seems to be true only for the ratio of the two longer dimensions l/w and less important for the other ratios w/h and l/h.

7. Discussion

Examples of room aspect ratios from the literature and some interesting cases from the present analysis are collected in Table 1 and rank-ordered from best to worst. The optimum ratios in the top are those from Meissner (2018) labelled A, B, and C and one of the optima from Walker (1993) that is very close to the A case.

Case D is the additional local optimum identified in one of the red regions in Fig. 3. However, we need to be very cautious about this because it describes a very flat room. If the volume is 300 m^3, the height is only 3.1 m, and for volumes below 200 m^3, the height is less than 2.7 m. In rooms for singing or playing musical instruments, the height must be sufficient to avoid tonal coloration, and in a rehearsal room, the height is important for a good blend of the sound from more instruments. For that reason, case D cannot be recommended for music rooms. It may work fine for a dance studio and other rooms where singing or playing of musical instruments is not the top priority.

Next in the list in Table 1 is the case corresponding to (3:4:5), which was the optimum found by Bolt (1946). The ratio of (1:1.26:1.59) recommended by Volkman (1942) is also good, whereas one of his alternative recommendations (1:1.59:2.52) does not perform well. The golden ratio is sometimes claimed to be a good choice for sound studios, but it comes rather low on the list and thus cannot be recommended.

At the bottom of the list are the cube and two variations of the cube, namely the "half cube" (1:2:2) that is a flat room with the height equal to half the height of the cube, and the "double cube" (1:1:2) that is a long room with the length twice the length of the cube.

The examples listed in Table 1 reveal some unexpected findings. Although the cubical room is in the bottom of the list, the double cube is even worse. The explanation is that in the double cube, length and width are bigger and thus the first natural frequencies are lower and the frequency intervals are larger than in the cube with the same volume.

Another surprise is the case (1:1.45:2), which is labelled $*$ in the table. This case has $\psi(25) = 1.65$ and is ranked as equally good as the Volkman 1 optimum, even though the length-to-height ratio is 2. The next case on the list labelled ** is also quite good, even though the length-to-height ratio is 3.

To understand these results, we need to look more closely at the two ratios w/h and l/w. The best rooms have l/w within the range $1.15 < l/w < 1.45$. That includes the two rooms with l/h equal to 2 and 3. The rooms that perform badly have either $l/w < 1.15$ or $l/w > 1.45$. This observation points at l/w as an important parameter for aspect ratios of rooms. The l/w ratio may be the most important single parameter for evaluating acoustical quality of small rooms. The

TABLE. 1. Selected examples of room aspect ratios in rank order from best to worst, using the FSI as acoustical quality measure.

Label	w/h	l/h	l/w	FSI	Quality
A	1.20	1.45	1.21	1.33	Best
Walker 1	1.19	1.40	1.18	1.36	
B	1.40	1.89	1.35	1.51	
C	1.48	2.12	1.43	1.54	
D	2.75	3.57	1.30	1.56	
Bolt	1.33	1.67	1.25	1.59	
Volkman 1	1.26	1.59	1.26	1.65	
*	1.45	2	1.38	1.65	
**	2.30	3	1.30	1.68	
Walker 2	1.75	2.20	1.26	1.78	
Volkman 2	1.59	2.52	1.59	1.81	
Golden ratio	1.618	2.618	1.618	1.94	
	2	3	1.5	2.16	
Cox and D'Antonio (bad)	1.075	1.868	1.738	2.39	
	2	4	2	2.70	
	1	4	4	2.74	
	1	3	3	3.00	
Louden (bad)	1.4	2.8	2	3.01	
Half cube	2	2	1	3.28	
Cube	1	1	1	3.71	
Double cube	1	2	2	3.91	Worst

w/h ratio, on the other hand, seems to have very little influence on the results. In Fig. 3, it is seen that it is important to keep $w/h > 1$, but little happens around $w/h = 2$ and $w/h = 3$.

Looking at the three very good aspect ratios labelled A, B, and C and marked in Fig. 3, they are approximately located on a straight line. The linear regression line through the points has the formula

$$l = 2.3558 \cdot w - 1.3838 \cdot h. \tag{3}$$

Another observation, also seen in Table 1 is that the three optimum aspect ratios A, B, and C are characterized by $l/w \approx w/h$. This implies that w is approximately the geometrical average of l and h and that the volume $V = l{\cdot}w{\cdot}h \approx w^3$. More precisely, it is found that

$$w \cong 1,007 \cdot \sqrt[3]{V}. \tag{4}$$

This relationship is depicted in Fig. 4. The figure also presents graphs for the height and length as functions of volume for each of the three optimum aspect ratios. It is seen that at 300 m^3, the length is between 8 and 10 m, and the height is between 4.5 and 5.5 m.

8. Suggested application for room design

For music rehearsal rooms and studios where good acoustics has high priority, one of the three best aspect ratios A, B, or C should be strived for. In addition to the three dimensions of the room, the required volume is a parameter. Fig. 4 may be helpful in finding solutions for volume and room dimensions.

In a practical situation, it is often so that the height of the room is fixed. As an example, we set $h = 4.5$ m. Then it is possible to meet one of the optimum aspect ratios with volumes of either 155, 240, or 280 m^3 (see Fig. 4). In the latter case, we read from the figure that the dimensions should be 4.5, 6.5, and 9.5 m. A different application of Fig. 4 is that the desired volume is given. Then the graphs can be used to find which room dimensions will meet one of the optimum aspect ratios. For example, a 50 m^3 room should have height between 2.5 and 3.1 m. For any of the three optimum aspect ratios, the width is linked to the volume as in Eq. (4).

If the acoustical requirements are not very strict, the aspect ratios can be chosen near the optima by using the regression line in Eq. (3). A reasonably smooth frequency response is ensured when the FSI is $\psi(25) \leq 1.75$, which is the case for any point at the line if w/h is within 1.1–1.6.

With reference to Fig. 3, a very simple design rule is $1.15 < l/w < 1.45$ and $w/h > 1.1$. In addition, a value of w/h close to 2 should be avoided. For rooms with less demanding acoustics, it may be sufficient to require $1.1 < l/w < 1.6$.

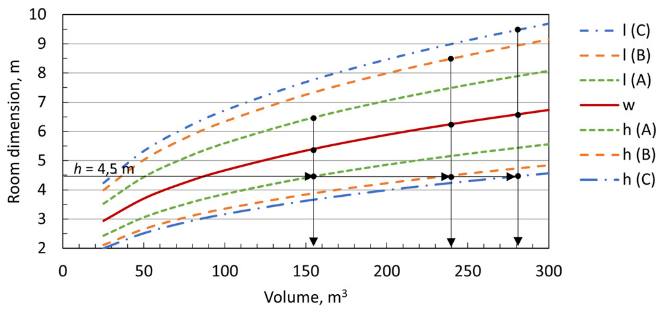

Fig. 4. Relation between volume and length, width, and height for the optimum aspect ratios A, B, and C. The black arrows show as an example three alternative volumes for a room height (h) of 4.5 m and the corresponding widths (w) and lengths (l).

An important finding is that the length-to-width ratio should not exceed 1.6. However, this applies to small rooms only. For rooms with volume much larger than 300 m^3, the lowest room modes are well below 20 Hz, and the frequency spacing of the modes is not a relevant acoustical parameter. The classical concert halls with shoe-box shape often have length-to-width ratios of 2 or more, which is natural because the audience area occupies a major part of the floor area, and this aspect ratio is not a problem in such large rooms.

9. Conclusion

The influence of the aspect ratio of room dimensions on the frequency distribution of the room modes has been studied using the relative variance of the frequency intervals as a criterion for the acoustical quality. The study deals with small rooms limited to a maximum of 300 m^3. Assuming that the length and width are greater than the height, it is found that the length-width ratio is much more important than the width-height ratio. The length-width ratios 1 and 2 are very bad and should be avoided. On the other hand, the ratio of 2 between the length and height or between the width and height has only minor influence. It is found that the length-width ratio should be within 1.15–1.45 to get a reasonably good frequency spacing of the room modes. The height can be chosen more freely without compromising the acoustical quality. Only the width-to height ratio should be greater than 1.1, and in rooms for singing or playing musical instruments, the height must be sufficient to avoid tonal coloration and ensure a good blend of the sound. The findings apply to small rooms for music practice and rehearsal, sound studios, control rooms, etc.

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