ASSESSMENT OF WAVE-BASED METHODS FOR ROOM ACOUSTIC SIMULATIONS

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1 INTRODUCTION

Room acoustic simulations performed with wave-based methods are becoming feasible alternatives to geometrical acoustics thanks to the increasing power of computers. In this preliminary study two of these methods are investigated: the Finite Difference in the Time Domain (FDTD) and the Equivalent Source Method (ESM). The Image Source Method (ISM) is also used as a reference for the comparison of results. The two methods chosen perform calculations in the time domain. This has the advantage of obtaining results for all frequencies up to a selected limit at once instead of running a simulation for each individual frequency needed. Time-domain calculations also directly yield the impulse response of a room, from which can be derived the acoustic parameters and auralization signals.

2 THEORY

2.1 Image Source Method

A two-dimensional rectangular domain with rigid boundaries is considered in the present study. The analytical solution describing an impulse response in this domain can be obtained with the Image Source Method[1]. For a given point source in the domain, the ISM models sound reflections by placing image sources at its symmetrical positions with respect to the boundaries. The placement of sources at symmetrical positions is then applied to the newly found image sources and repeated until a sufficient order of image sources is reached. The signals of the image sources are determined to satisfy the boundary conditions. In the case of rigid boundaries, i.e. no particle velocity in the direction perpendicular to the boundaries, the image sources emit the same signal as the source in the domain. Eventually, the sound pressure p at a receiver point r is calculated by adding up the contributions from all the sources:

$$p(r,t) = \sum_{n=1}^{N} p_n(r_n, r, t) \tag{1}$$

with n the index of the source, r_n its position, and p_n its contribution. The distance from a source n to the receiver is $R_n = |r - r_n|$. Point sources in 2D are represented as cylindrical sources, and their contributions are thus calculated based on cylindrical radiation and free field propagation. The sound pressure radiated by a cylindrical source is described by the Hankel function, which asymptotic behaviour is proportional to $1/\sqrt{R_n}$ when the distance R_n goes to infinity[2]. In this study, it is approximated that the radiated sound pressure is a function of $1/\sqrt{R_n}$ for all source-receiver distances. The sound pressure radiated from a cylindrical source at a receiver is then

$$p_n(r_n, r, t) = \frac{g(t - \tau_n)}{\sqrt{R_n}} \tag{2}$$

where $g(t)$ is the source signal and $\tau_n = R_n/c$ is the travel time between the source and the receiver.

2.2   Finite Difference in the Time Domain

Finite Difference schemes are well-known methods to solve differential equations numerically[3]. When applied to acoustic problems, the conservation of mass and conservation of momentum are considered:

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\rho}\boldsymbol{\nabla} p \tag{3}$$

$$\frac{\partial p}{\partial t} = -\rho c^2 \boldsymbol{\nabla} \cdot \boldsymbol{v} \tag{4}$$

The sound pressure is noted p, the particle velocity v, ρ is the air density, and c is the speed of sound. In FDTD, the derivatives are approximated with finite differences. In the present case, first order central difference in space and forward difference in time are used. To do so, the domain is divided into cells forming a Cartesian grid. The centres of cells are pressure nodes, and velocity nodes are placed at the middle points between two adjacent pressure nodes. Velocity nodes are also located on the boundaries. The pressure and velocity grids are staggered by half a step in time to enable proceeding of the method. As notation, i and j represent the space indexes in the x and y directions respectively, k is the time index, h is the spatial step, and Δt is the time step. The resulting update equations for the sound pressure and particle velocities in the x and y directions are

$$p(x_i, y_j, t_k) = p(x_i, y_j, t_{k-1}) - \rho c^2 \frac{\Delta t}{h}\left(\begin{array}{l} v_x\left(x_{i+\frac{1}{2}}, y_j, t_{k-\frac{1}{2}}\right) - v_x\left(x_{i-\frac{1}{2}}, y_j, t_{k-\frac{1}{2}}\right) \\ + v_y\left(x_i, y_{j+\frac{1}{2}}, t_{k-\frac{1}{2}}\right) - v_y\left(x_i, y_{j-\frac{1}{2}}, t_{k-\frac{1}{2}}\right) \end{array}\right) \tag{5}$$

$$v_x\left(x_{i+\frac{1}{2}}, y_j, t_{k+\frac{1}{2}}\right) = v_x\left(x_{i+\frac{1}{2}}, y_j, t_{k-\frac{1}{2}}\right) - \frac{1}{\rho}\frac{\Delta t}{h}\left(p(x_{i+1}, y_j, t_k) - p(x_i, y_j, t_k)\right) \tag{6}$$

$$v_y\left(x_i, y_{j+\frac{1}{2}}, t_{k+\frac{1}{2}}\right) = v_y\left(x_i, y_{j+\frac{1}{2}}, t_{k-\frac{1}{2}}\right) - \frac{1}{\rho}\frac{\Delta t}{h}\left(p(x_i, y_{j+1}, t_k) - p(x_i, y_j, t_k)\right) \tag{7}$$

The spatial and time steps in FDTD are restricted to ensure that the algorithm produces a stable solution. In the present 2D case, the stability condition is

$$c\Delta t \leq \frac{h}{\sqrt{2}} \tag{8}$$

2.3   Equivalent Source Method

The ESM is mostly used for radiation and scattering problems, but it can also be suited for the calculation of reflections in a room[4]. It consists in placing a set of sources outside the domain and adjusting their strengths so that the boundary conditions are satisfied. The rigid boundary condition of the considered domain corresponds to the equation

$$\boldsymbol{\nabla} p_r \cdot \boldsymbol{n} = -\boldsymbol{\nabla} p_i \cdot \boldsymbol{n} \tag{9}$$

where p_i and p_r are respectively the incident and reflected sound pressures, and $\boldsymbol{n}$ is the normal vector to the boundary pointing inwards. For boundary points situated in the corners, the normal vector is taken with a 45° angle to both adjacent sides of the rectangle. The reflected sound pressure is composed of the contributions of all equivalent sources, similarly to equation (1) for the ISM. For one source-receiver pair, in this case an equivalent source and a boundary point, the sound pressure p_n at the receiver was expressed in equation (2). Noting the unit directional vector from the source to the receiver $\boldsymbol{u_n}$, the gradient of the sound pressure can be written

$$\nabla p_n = \frac{\partial p_n}{\partial r_n} \boldsymbol{u_n} \tag{10}$$

$$\nabla p_n = \left(-\frac{g(t - \tau_n)}{2R_n^{3/2}} - \frac{1}{c\sqrt{R_n}} \frac{\partial g}{\partial t} \right) \boldsymbol{u_n} \tag{11}$$

The time derivative of g can then be approximated with a 1st order finite difference. Following this, the gradients in equation (9) are replaced with equation (11) and continuous time is turned into discrete steps. At each time step k, the problem can now be expressed in matrix form as

$$A\boldsymbol{q_k} = B \tag{12}$$

$\boldsymbol{q_k}$ is the vector of unknown equivalent source strengths at the time step k, A is a matrix representing the propagation of sound from the equivalent sources to the boundary points, and B is a matrix describing the known sound field at the boundary points. Because the problem in equation (12) has to be solved at every time step, B is made of both the incident sound field and updated contributions of the equivalent sources from previous time steps. Once the source strengths have been determined for all time steps, the sound waves are propagated to receiver points located in the domain so as to obtain the desired sound pressure.

A recommendation from Dunn and Tinetti indicates that the equivalent sources should be placed on a 90% replica of the scattering object for an exterior domain[5]. It is here adapted to the interior domain by placing the equivalent sources on a 110% replica of the rectangular room.

3   NUMERICAL SETUP

The two-dimensional domain consists in a rectangle centred on the origin of the x-y plane, with a total length of 4 m in the x direction and a total width of 2 m in the y direction. A source is arbitrarily placed at $x = -1$ m and $y = 0.667$ m. It emits a sine-modulated Gaussian pulse, defined as

$$g(t) = -Ae^{-\frac{(t-t_0)^2}{2\sigma^2}} \sin\left(2\pi f_0(t - t_0)\right) \tag{13}$$

with the amplitude $A = 0.1$, the zero of the pulse $t_0 = 1$ ms, the bandwidth parameter $\sigma = 2.10^{-4}$, and the center frequency $f_0 = 200$ Hz. The total time length of the simulations is set to $T = 0.02$ s. The source signal and its Fourier transform are shown in Figure 1. In the time domain, the pulse reaches a peak and then a dip of equal amplitude before smoothly dying out. In the frequency domain, the pulse is equivalent to a bandpass filter.
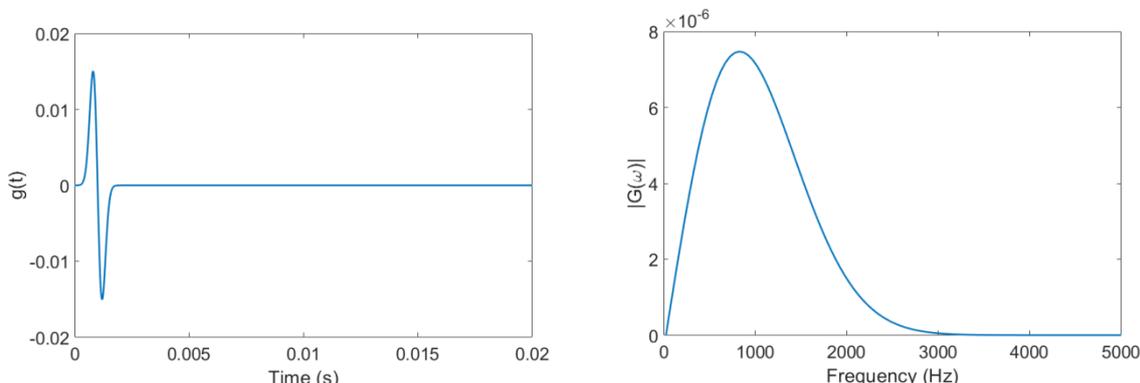
(a) Time signal.   (b) Frequency content.
Figure 1: Sine-modulated Gaussian pulse emitted by the source.

The FDTD mesh is designed to perfectly fit the domain by setting the number of cells in the y direction, corresponding to the width of the rectangle. In order to study the convergence of the method, this number is defined as $N_y = 3^\beta$, β ranging from 1 to 4. The sound pressure at the source point is bound to follow the source signal until the sine-modulated Gaussian pulse dies out. This limit is set to $g(t_s) > -10^{-5}$. After this time t_s, the source point is released and the sound pressure is calculated according to equation (5). The sampling frequency for the FDTD calculations is chosen to be $f_s = 48 \, \text{kHz}$, resulting in the stability condition being $h \geq 1.01 \cdot 10^{-2} \, \text{m}$ according to equation (8).

For the ESM, the sound pressure is calculated on a fixed receiver grid corresponding to the pressure nodes of the FDTD grid with $N_y = 3^3$. The number of surface points is of the order $O_\gamma = 15 \cdot 3^{\gamma-1}$, γ ranging from 1 to 6, and the exact value is chosen to be the closest integer number fitting the domain. The sampling frequency for this method is set to $f_s = 10 \, \text{kHz}$.

The error between the simulation results and the reference method is measured by averaging the error over time steps and receivers:

$$e = \sum_{i=1}^{N_r} \sum_{k=1}^{N_t} \frac{\left|p_{ref}(r_i, t_k) - p(r_i, t_k)\right|}{N_r N_t} \tag{14}$$

In this equation, N_r is the number of receivers, N_t is the number of time steps, p_{ref} is the reference sound pressure from the ISM, and p is the calculated sound pressure.

4   RESULTS AND DISCUSSION

The sound pressure generated in the domain by the source is calculated with the different methods presented. Convergence rates are plotted in Figure 2 by running the simulations with different spatial resolutions h^{-1}. In the case of the FDTD, the spatial resolution dictates the spacing in the grid. The finite difference approximation used is of order 1, hence the algorithm should also converge with order 1. As can be seen from the figure, the FDTD does not converge below $h^{-1} = 10 \, \text{m}^{-1}$ when the calculation grid is too coarse. Once the grid is fine enough, the convergence rate appears to be close to linear. However, more data points to confirm this trend could not be acquired because of the stability limit of the method. For the ESM, the spatial resolution governs the distance between boundary points. A 1st order finite difference approximation is also used, and it could be expected to find the same order in the convergence rate. The results obtained indicate that the method does not converge when the spatial resolution increases. An explanation to this behaviour is found in Figure 3, which shows the condition number of the matrix A from equation (12) for the different values of h^{-1}. It is observed that the condition number increases drastically with h^{-1}, which means that the matrix is near-singular. Consequently, the matrix inversion needed to solve the problem is very sensitive to small perturbations and can yield large errors.
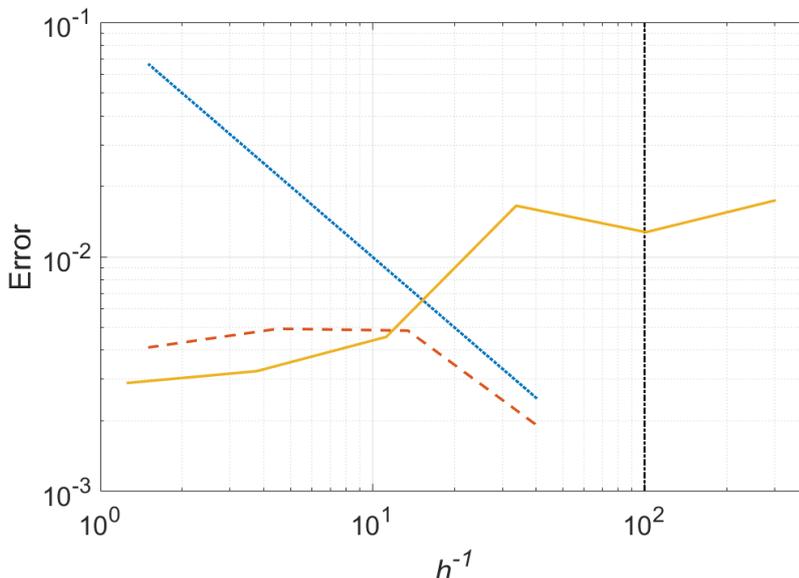
Figure 2: Convergence rates of the methods. Dotted blue line: $O(h)$; Black dash-dotted line: stability limit for FDTD; Dashed red line: results from the FDTD; Solid yellow line: results from the ESM
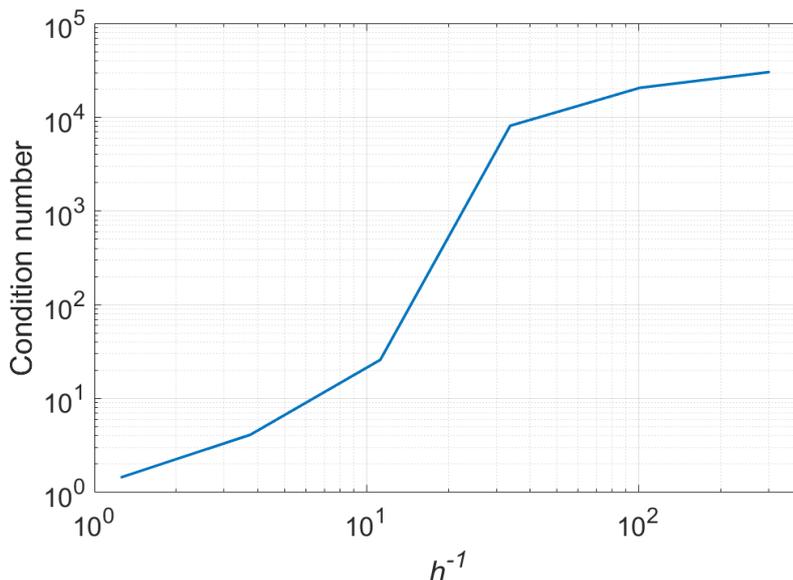

Figure 3: Condition number of the matrix A in the ESM

As expected, the FDTD behaves nicely when the calculation grid is fine enough, and the simplicity of the algorithm ensures efficient computations. Also, because it is a well-known method, solutions exist to most of the challenges that can be encountered, such as dispersion problems or the cartesian grid not fitting room geometries. Nevertheless, there exist some inherent drawbacks to this method. Firstly, a high value is required for the sampling frequency in order to keep the calculations stable with a high spatial resolution. Secondly, room acoustic problems usually focus on a limited number of receiver positions in the room. Therefore, there can be some overcalculation in FDTD as a grid covering the whole domain is required for the algorithm. The ESM is a physically based method that has been proven efficient for scattering problems. Changing its formulation from

an exterior to an interior problem is expected to return reliable results, even though it is not the case in this preliminary study. Indeed, solutions to the high condition number of the matrix that was encountered exist. Some of these solutions include a singular value decomposition of the matrix or a low-pass filtering of the results for example. It should thus be investigated further how the ESM can perform compared to the FDTD.

5   CONCLUSION

The FDTD and ESM are two potential wave-based methods to be implemented for room acoustic simulations. This preliminary study exposed different issues for these methods. On one hand, the FDTD seems to be not well suited to room acoustics with the need for a grid over the whole domain and a sampling frequency far above the range of interest. On the other hand, the ESM exhibited convergence issues caused by the inversion of an ill-conditioned matrix in its algorithm. However, these issues can be overcome with already existing techniques. Therefore, both the FDTD and the ESM remain credible solutions for room acoustic simulations at low frequencies.

6   REFERENCES

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