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# Combining image and equivalent sources for room acoustic simulations

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**Abstract**

Room acoustic simulation tools based on geometrical acoustics fail to predict accurately the behaviour of rooms at low frequencies. This inaccuracy can lead to poor acoustic conditions especially in rooms of moderate volume, such as classrooms and meeting rooms. The present paper tackles this problem by introducing a simulation technique for room acoustics based on the Image Source Method and the Equivalent Source Method. These methods are implemented in the time domain to achieve efficient calculations with reliable results for low to medium frequencies. Given a source emitting a pulse signal in a room, image sources up to different orders are defined to determine early reflections. With the direct sound field from the original source and the contributions of the image sources, the remaining part of the sound field is solved by the Equivalent Source Method to account for late reflections and diffraction effects. The results of the test simulations carried out show that this combined Image-Equivalent Source Method is a suitable option for room acoustics.

Keywords: Room Acoustic Simulation, Equivalent Source Method, Image Source Method

## 1 INTRODUCTION

The current state of room acoustic simulation is well-suited for mid to high frequencies or large volumes, such as concert halls and auditoria, thanks to the principles of geometrical acoustics. Geometrical acoustics methods are indeed reliable in these conditions and computationally efficient, but they fail at producing accurate results at low frequencies. Wave-based methods are the most common solution in acoustics for simulating low frequencies, with the Finite Element Method and the Boundary Element Method being the most popular. However, they have the disadvantage of being computationally heavy and thus unpractical for room acoustics. The Equivalent Source Method (ESM), however, is a compromise solution. Similarly to the Boundary Element Method, it simulates the sound field by placing sources around the boundaries of the domain, but the main difference lies in the approximations used to reduce the calculation load. It is therefore an interesting candidate for simulating room acoustics at low frequencies. Furthermore, it is possible to easily combine the ESM with the Image Source Method (ISM) to enhance its accuracy at a low computational cost. Calculations performed in the time domain also allow to limit computation time since one simulation is enough to cover the whole frequency range considered.

The Equivalent Source Method is typically used to calculate sound radiation or scattering from objects. Kropp and Svensson [1] laid the foundation of the ESM in the time domain for acoustic problems, and Lee, Brentner and Morris [2] furthered its study. Recently, Lee [3] gathered the state-of-the-art knowledge on the topic and established guidelines to solve radiation and scattering problems. The present study is the first step towards a different approach of solving interior problems with the ESM, where the sound field inside a room will be simulated rather than around an object.

## 2 THEORY

The sound reflection on a single plane with rigid boundaries will be investigated in this paper. In all the simulations considered, sound propagates freely from sources to receivers in a three-dimensional domain. This propagation is described by the free field Green's function in 3D:

$$G(r,t) = \frac{1}{4\pi}\frac{\delta(t-r/c)}{r}\,, \tag{1}$$

with $r$ the distance between a source and a receiver, $c$ the speed of sound, and $\delta$ the Dirac function. Given a source with a prescribed volume velocity $q(t)$, the sound pressure at a receiver is obtained after multiplication with the Green's function and integration over time:

$$p(r,t) = \frac{1}{4\pi}\frac{q(t-r/c)}{r}\,. \tag{2}$$

When several sources emit at the same time, their individual sound pressures are summed up to obtain the total sound pressure at the receiver.

The Equivalent Source Method allows to calculate the sound field inside or around an object by solving the boundary condition equation. To do so, the plane considered in this study is discretized with control points. With the sound source on one side of the plane, a virtual surface with same dimensions is created on the other side and discretized with equivalent sources. The number of equivalent sources can be lower than that of surface points to reduce computation time, leading to an overdetermined problem. The volume velocities of these sources are then determined so that the sum of the incident sound field from the original source and the reflected sound field from the equivalent sources satisfies the boundary conditions on the control points. In this paper only rigid boundaries are considered. In terms of sound pressure, it is equivalent to

$$\overrightarrow{\nabla} p_r \cdot \overrightarrow{n} = -\overrightarrow{\nabla} p_i \cdot \overrightarrow{n}\,, \tag{3}$$

with $p_i$ the incident sound pressure, $p_r$ the reflected sound pressure, and $\overrightarrow{n}$ the unit normal vector pointing towards the original sound source. If the volume velocity time signal of the original source is known as an analytical and derivable formula, the gradient of the incident pressure can be calculated directly. The reflected sound pressure, however, is related to the volume velocities of the equivalent sources that need to be determined. By noting $q_e$ the volume velocity of the equivalent source $e$ and using equation (2), the reflected pressure gradient is expressed as

$$\overrightarrow{\nabla} p_r = \frac{1}{4\pi}\sum_e \left( -\frac{q_e(t-r_e/c)}{r_e^2} - \frac{1}{cr_e}\frac{\partial q_e}{\partial t}(t-r_e/c) \right) \overrightarrow{u_{r_e}}\,. \tag{4}$$

In this equation, $r_e$ represents the distance and $\overrightarrow{u_{r_e}}$ the unit directional vector from the equivalent source $e$ to the surface point under consideration. Numerical methods, including the ESM, work in a discrete time domain. In order to represent $t - r_e/c$ in discrete time, the volume velocities $q_e$ are discretized assuming linear interpolation; hence, their derivatives are approximated with first order. With $\tau_e = t - r_e/c$, the neighbouring discrete time steps are noted $t_e^-$ and $t_e^+$ such that $t_e^- \leq \tau_e \leq t_e^+$. Applying the time discretization to equation (4) with a time step $\Delta t$, the boundary condition in equation (3) becomes

$$\frac{1}{4\pi\Delta t}\sum_e \left( -\frac{\tau_e - t_e^-}{r_e^2} - \frac{1}{cr_e} \right) q_e(t_e^+)\overrightarrow{u_{r_e}} \cdot \overrightarrow{n} = -\overrightarrow{\nabla} p_i(\tau_e) \cdot \overrightarrow{n} - \frac{1}{4\pi\Delta t}\sum_e \left( -\frac{t_e^+ - \tau_e}{r_e^2} + \frac{1}{cr_e} \right) q_e(t_e^-)\overrightarrow{u_{r_e}} \cdot \overrightarrow{n}\,. \tag{5}$$

It appears that the volume velocities of the equivalent sources at a new time step can be determined from previous time steps, resulting in a marching algorithm. The system made of the equations for all surface points can be written in matrix form and solved at once with the least squares method for each time step.

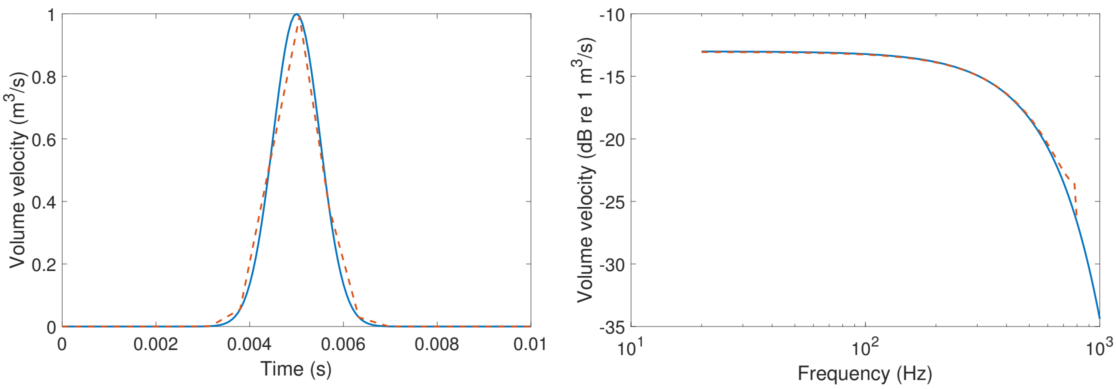


Figure 1. Source signal in the time domain and its frequency content. Solid blue line: $f_s = 16000\,\text{Hz}$; Dashed red line: $f_s = 1600\,\text{Hz}$.

The Image Source Method also relies on solving the boundary condition equation. Given a source in front of a single plane, an image source is located in the symmetric position relative to the plane. The signal of the image source is then adjusted to fulfill the boundary conditions. In the case of rigid boundaries, the image source simply emits the same signal as the original source.

In order to combine image and equivalent sources in one method, the image source for the single plane is first implemented. Its contribution is added to that of the original source to form the incident sound pressure, which is then incorporated in the ESM algorithm. This will later on be called the Image-Equivalent Source Method (IESM).

The Edge Diffraction Method [4] is based on the theory from Biot and Tolstoy [5] about reflection around an infinite rigid wedge. The sound field is decomposed between incident wave, specular reflection, and diffraction waves with an opposite polarity emanating from the edges of the scattering object.

## 3 NUMERICAL SETUP

The simulation setup consists of a single plane with rigid boundaries that extends from $-2.5\,\text{m}$ to $4.5\,\text{m}$ in the $x$-direction and from $-3\,\text{m}$ to $4\,\text{m}$ in the $z$-direction. A point source is located at the coordinates $(-1\,\text{m},\ 1.5\,\text{m},\ 1\,\text{m})$ and emits a Gaussian pulse which volume velocity is

$$q_i(t) = e^{-\frac{(t-t_0)^2}{2s^2}}\,. \tag{6}$$

$t_0$ represents the center time of the pulse and $s$ is the standard deviation controlling the width of the pulse in the time domain. These parameters are set to $t_0 = 5\,\text{ms}$ and $s = 0.5\,\text{ms}$. This pulse was chosen in order to limit the frequency bandwidth of the signal used in the simulations. The corresponding source signal and its frequency content are shown in Figure 1 with two different sampling frequencies $f_s$. By taking $3\,\text{dB}$ below the maximum amplitude, the cut-off frequency of the pulse is found to be $400\,\text{Hz}$. The sampling frequency is thereafter set to $1600\,\text{Hz}$, limiting the simulations to frequencies below $800\,\text{Hz}$.

Firstly, the growth rate of computation time in the ESM relative to the size of the problem is investigated. Different simulations are performed with varying spacing between surface points. The spacing between equivalent sources is set 4 times larger than between surface points, meaning that each source is centered between

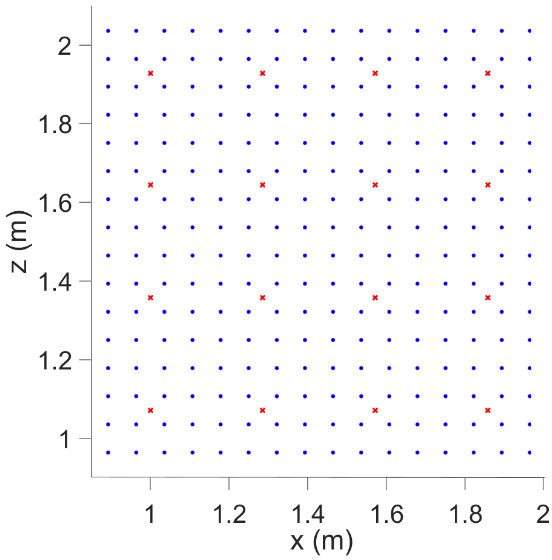


Figure 2. Close up view of the plane discretization. Blue dots: surface points; Red crosses: equivalent sources.

16 points, as seen in Figure 2. The distance between the surface plane and the virtual plane comprising the equivalent sources is 11.4 mm. The single plane reflection being a very simple case, the residual of equation (5) is nearly constant in all simulations. Thus, a convergence study is not performed here.

Secondly, the sound pressure at three receivers with coordinates $(2\,\mathrm{m},\ 2\,\mathrm{m},\ 1\,\mathrm{m})$, $(-3.5\,\mathrm{m},\ 1.5\,\mathrm{m},\ 1\,\mathrm{m})$ and $(-5\,\mathrm{m},\ 1\,\mathrm{m},\ 1\,\mathrm{m})$ is investigated. This placement is chosen so that the reflection points corresponding to those receivers are respectively located on the plane, on the edge of the plane, and outside the plane. In the ESM the plane surface is discretized with control points spaced by 71.5 mm, corresponding to one sixth of the wavelength for the highest frequency. In the ISM only one image source is required for the single plane case, but this method has the drawback of assuming an infinitely large plane where diffraction cannot be represented. Concerning the IESM simulation, the same parameters are applied as with the ESM and ISM taken separately. The Edge Diffraction Method calculates the response to a unit Dirac pulse. This impulse response is obtained in the time domain with only the first diffraction order, and it is then convoluted with the Gaussian pulse in equation (6).

## 4 RESULTS AND DISCUSSION

Although the single plane reflection case with rigid boundaries does not allow to study convergence, the computation time of the Equivalent Source Method can be investigated. Of particular interest is the way computation time increases with the size of the problem to be solved. Here, the size of the problem is varied by changing the spacing between control points on the surface, hence modifying the density of surface points $h$. The ratio between surface points and equivalent sources being fixed, the spacing of equivalent sources also varies accordingly. The result of this investigation is presented in Figure 3. The computation time of the ESM is seen to be proportional to the square of the surface point density, therefore it grows with second order. This means that refining the discretization of the surface would be attractive if the residual in equation (5) converges with order 2 or higher. As previously mentioned, it is unfortunately not possible to verify this convergence rate with the present test case.

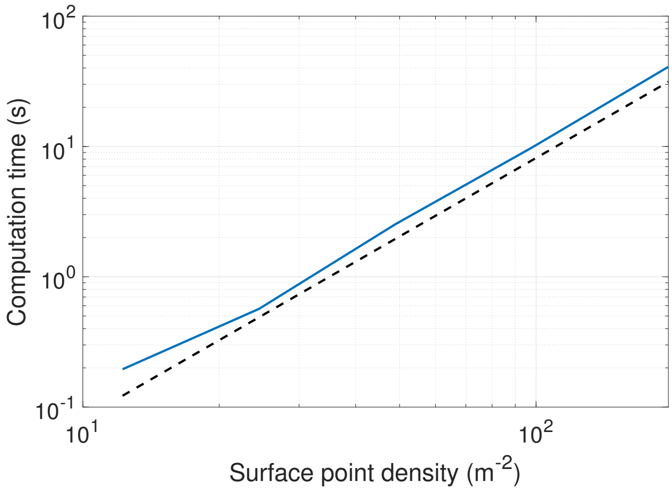


Figure 3. Evolution of the computation time of the ESM according to the surface point density $h$. Solid blue line: computation time of the simulations; Dashed black line: $O(h^2)$.

The sound pressure at the receivers is now considered. For the first receiver, the reflection point is located on the plane. Consequently, the specular reflection should be the same as for an infinite plane, which is the case simulated with the ISM. This is in agreement with the results in Figure 4. Indeed, the direct sound and specular reflection is the same for all the methods used. However, discrepancies arise for the modelling of the diffraction that reaches the receiver between 0.03 s and 0.04 s. The ESM and the Edge Diffraction Method show a fair agreement in the sound pressure, even though the ESM seems to underestimate the amplitude of the diffraction. Besides, it appears clearly here that the waves emanating from the edges are in antiphase with the specular reflection. With the Image-Equivalent Source Method, the result obtained is identical to the ISM. This has a simple explanation: the sound field emitted by the image source satisfies the boundary condition in equation (3), therefore the equivalent sources remain inactive in the simulation. In order to account for diffraction effects in this case, the single plane should be modelled with a finite thickness. The equivalent sources would then be activated to fulfill the boundary conditions on all faces of the plate. The diffraction effect being absent of IESM simulations in this single plane case, this method will not be considered for the other two receivers.

The next receiver position was chosen so that the reflection point is located on the edge of the single plane. It implies that the specular reflection and the diffraction wave coming from that edge reach the receiver simultaneously. As seen previously, the specular and diffraction reflections are in antiphase, thus desctructive interference occurs. Given the size of the plane, this applies to the whole frequency range and the amplitude of the resulting reflection is expected to be half of what would be observed for an infinite plane [6]. This is what is observed in Figure 5 where the specular reflection in the Edge Diffraction Method is half as strong as in the ISM. However, the reflection of the ESM is not as attenuated. This is a confirmation that the ESM underestimates the amplitude of the diffraction. The sound pressure at the third receiver yields the same observations. The location of this receiver makes the reflection point to be situated outside the plane. Naturally, it is seen that the specular reflection is even more attenuated in this case, although not as much for the ESM as for the Edge Diffraction Method.

In these examples it was seen that the ESM is not as accurate as the Edge Diffraction Method. However,

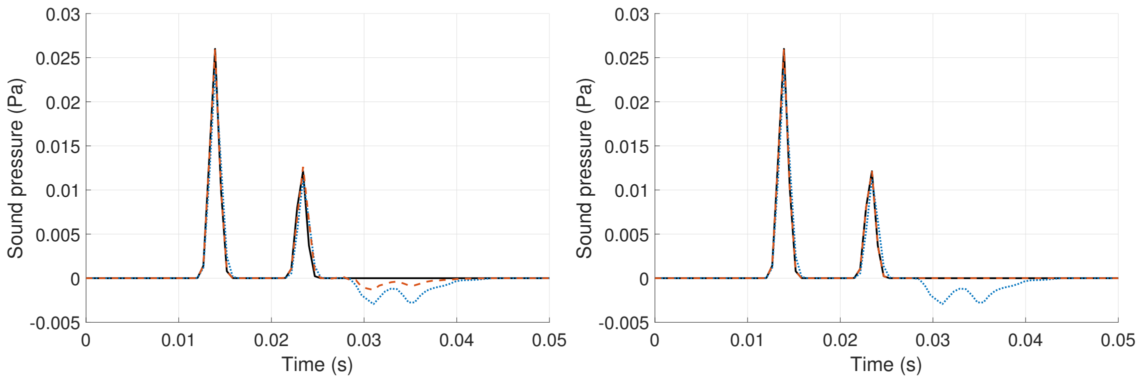


Figure 4. Sound pressure at the receiver with reflection point on the plane. Solid black line: Image Source Method; Dotted blue line: Edge Diffraction Method; Dashed red line: Equivalent Source Method (left) or Image-Equivalent Source Method (right).

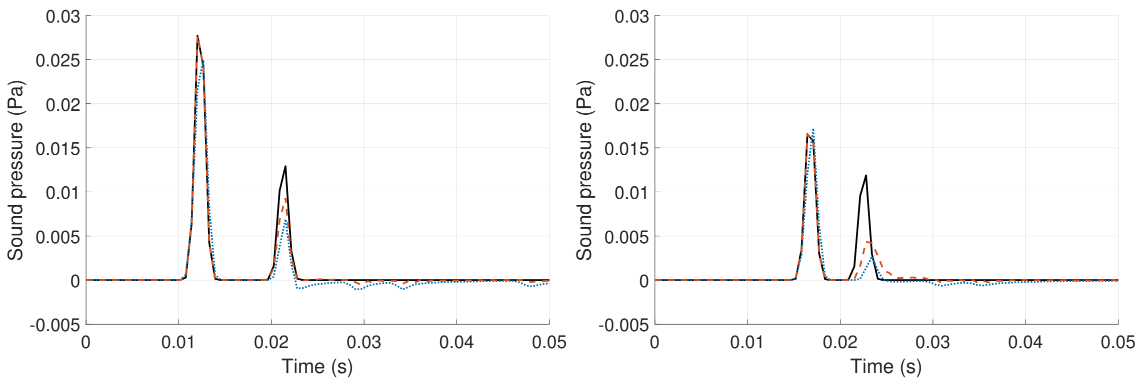


Figure 5. Sound pressure at two different receivers with reflection point on the edge of the plane (left) or outside the plane (right). Solid black line: Image Source Method; Dotted blue line: Edge diffraction Method; Dashed red line: Equivalent Source Method.

the accuracy is expected to be improved in the IESM. With the help of an image source accounting for the specular reflection, the equivalent sources will be devoted to model diffraction. Applying the IESM to a closed room will also allow to avoid the problem encountered here where equivalent sources remain inactive. Another topic that was not covered in this paper and will require attention for room acoustic simulation is the description of boundary conditions. In realistic room cases, the boundaries have a finite impedance that is frequency-dependent. This changes the boundary condition equation in the time domain with additional terms depending on past events.

## 5 CONCLUSION

This paper investigated the performance of the Equivalent Source Method and the Image-Equivalent Source Method for a single plane reflection with rigid boundaries. The ESM was seen to be accurate to model the specular reflection but slightly underestimated the amplitude of diffraction effects. The computation time was also found to grow with second order, although a convergence study could not be carried out to complement this finding. The IESM, however, could not yield correct results due to the boundary conditions being fulfilled by the image source alone; hence, the diffraction could not be modelled. This problem can later be encountered in room acoustic simulations with hanging reflectors for example. In order to activate equivalent sources and correctly model diffraction, such reflectors would need to be given a finite thickness. It is expected that the IESM will be more accurate than the ESM with similar convergence rate and growth rate of computation time.

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