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A revised theory for the absorption of a rectangular absorber in an infinite rigid baffle

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ABSTRACT

The reverberation chamber method to measure a material's absorption coefficient has known limitations. In particular, the measured value can exceed unity, due to the finite size of the studied sample, which creates edge diffraction effects. These are not accounted for in classical theories of absorption. A revised theoretical approach is proposed to evaluate the absorption of sound by a finite-sized absorber flush-mounted in an infinite rigid baffle. The absorption, reflection and transmission of sound through the sample are governed by the material's surface impedance (dissipation of energy) and the sample's radiation impedance, which depends on its geometry. The maximum possible absorption is achieved in an "open-window" configuration, in which sound passes freely through an aperture of the same dimensions as the sample. In that case, the absorbed sound comprises entirely of the sound transmitted to the other side of the baffle. For an open window, the absorption coefficient drops at low frequencies and at large incidence angles, due to edge diffraction. Therefore, we propose to compare sound absorption by a given sample to the corresponding open-window configuration. Comparisons with experimental data show that the revised theory makes it possible to better predict the absorption coefficient measured in a reverberation chamber.

Keywords: Absorption coefficient, edge diffraction, open window

1 INTRODUCTION

The absorption coefficient of materials is a central aspect of room acoustic predictions, as it constitutes essential input data to acoustic models. A common way of measuring it is the reverberation chamber method (ISO 354) [1], which characterizes absorber samples under diffuse field incidence. However, many uncertainties have been identified in this method, including lack of diffuseness, finite size effects and poor reproducibility between laboratories [2].

In particular, the finiteness of the sample under test can lead to absorption coefficient values above 1, due to additional diffraction effects at the edges. ISO 354 recommends a test sample size between 10 and 12 m^2 in order to mitigate the problem, but even with such sizes, edge diffraction is still present and the absorption coefficient can still be above 1. In terms of theoretical calculation, this is explained with a statistical absorption coefficient, in which the incident power on the surface is calculated under an infinite size assumption. However, it is not clear whether such a calculated coefficient corresponds exactly to the measured data in a reverberation chamber. In room acoustic applications, e.g. in simulations, it is common to truncate such absorption coefficients to 1, which is a very coarse approach.

The absorption by finite-sized samples has been extensively studied. For instance, assuming that the absorber is flush-mounted in a rigid baffle, Thomasson defined a corrected absorption coefficient, in which the incident power is replaced by the available power [3]. As a result, Thomasson's coefficient cannot exceed 1. However, the case of maximum absorption proposed by Thomasson is unrealistic, because it requires a surface impedance equal to the complex conjugate of the radiation impedance at all angles. It is common to rely on a variational approach to characterize the sound field at the absorber surface, which results in an analogous electrical circuit [4].

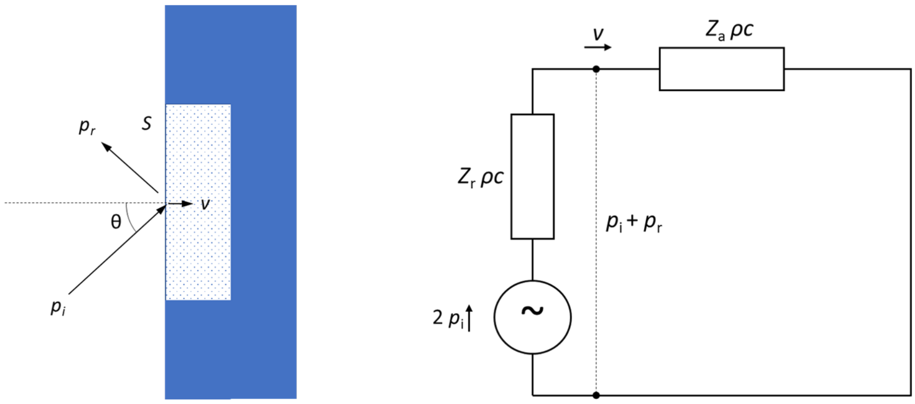

Figure 1. Reflection of a plane wave at oblique incidence on a finite-sized absorber flushed in a rigid backing. Left: sketch of the setup. Right: electrical analogue circuit.

In this paper, a revised theory for the absorption of finite samples is proposed, in which the absorber is compared to an ideal absorption case, corresponding to an open window of the same size of the absorber. Using such a reference for absorption was actually Sabine's original idea, as presented in his paper on reverberation ([6], p. 23-24).

2 THEORY

2.1 Setup and known equations

Figure 1 shows the studied setup. We assume a rectangular absorber of finite surface area S flush-mounted in an infinite rigid baffle. The sample is subject to an incident plane wave of amplitude p_i, impinging at an incidence angle θ. This results in a reflected wave of amplitude p_r, which is not a plane wave, unless the surface is infinite. The setup is commonly represented as an analogous electrical circuit, also shown in Figure 1. The analogy is valid under the assumption that the pressure over the surface behaves the same as the projection of the incident wave on the surface [4].

The quantities included in the model are the reflected pressure amplitude p_r, the air impedance ρc, the sample's surface impedance Z_a and its radiation impedance Z_r. In this paper, the impedances Z_a and Z_r are normalized to the air impedance ρc. In the analogy, the driving voltage is $2p_i$ and it corresponds to the sound pressure in front of the baffle only (without the absorber). The pressures are expressed at the surface of the absorber. The surface impedance is related to the absorption of sound by the specimen, and can be expressed from the sound field quantities as

$$Z_a = \frac{p_i + p_r}{\rho c v}. \tag{1}$$

The radiation impedance represents the effect of the absorber's size. It is expressed as follows,

$$Z_r = \frac{p_i - p_r}{\rho c v}. \tag{2}$$

The radiation impedance can be expressed as a quadruple integral equation depending on the absorber dimensions, but we make use of an approximation for easier implementation [5]. Z_r depends both on frequency and incidence angle.

For larger samples,

$$Z_r \approx \frac{1}{\cos(\theta)}. \tag{3}$$

This asymptotic value, which is a real number, corresponds to the radiation impedance of an infinite plane. The approximation is valid when the Helmholtz number ke is much larger than 1 (k being the wavenumber and e being related to the absorber's surface area by $e = 1/2\sqrt{S}$).

From this model, it is possible to study various relevant quantities. The complex reflection factor is expressed as

$$R(\theta) = \frac{p_r}{p_i} = \frac{Z_a - Z_r^*}{Z_a + Z_r}. \tag{4}$$

The angle-dependent absorption coefficient is

$$\alpha(\theta) = 1 - |R(\theta)|^2 = \frac{4\mathrm{Re}(Z_a)\mathrm{Re}(Z_r)}{|Z_a + Z_r|^2}. \tag{5}$$

The incident power on the surface can be expressed as a function of the radiation impedance,

$$P_{inc} = \frac{1}{2}\frac{|p_i|^2}{\rho c} S \frac{1}{\mathrm{Re}(Z_r)} = \frac{P_0}{\mathrm{Re}(Z_r)}, \tag{6}$$

where P_0 is a reference sound power quantity. When $ke >> 1$, a common approximation is to estimate the incident power as the projected power of the incident wave on the absorber's surface. This approximation is extensively used in the field of geometrical acoustics. In that case, the incident power becomes

$$P_{inc,\infty} = \frac{1}{2}\frac{|p_i|^2}{\rho c} S \cos(\theta) = P_0 \cos(\theta). \tag{7}$$

$P_{inc,\infty}$ corresponds to the incident power expressed in Eq. (6), using the approximation of Eq. (3) for Z_r. At normal incidence ($\theta = 0°$), we have $P_{inc,\infty} = P_0$. Therefore, P_0 corresponds to the incident sound power of a plane wave at normal incidence when disregarding finite-size effects. With Eq. (7), the incident power becomes zero at grazing incidence ($\theta = 90°$), which is not the case in reality. The absorbed power is found in the electrical circuit at the terminals of the Z_a impedance,

$$P_{abs} = 4P_0 \frac{\mathrm{Re}(Z_a)}{|Z_a + Z_r|^2}. \tag{8}$$

In a diffuse sound field, the incident power and the absorbed power must be integrated over the incidence angle. A random incidence absorption coefficient is then obtained,

$$\alpha_{rand} = \frac{\Pi_{abs}}{\Pi_{inc}} = \frac{\int_0^{\frac{\pi}{2}} \frac{4\mathrm{Re}(Z_a)}{|Z_a+Z_r|^2} \sin(\theta)\mathrm{d}\theta}{\int_0^{\frac{\pi}{2}} \frac{1}{\mathrm{Re}(Z_r)} \sin(\theta)\mathrm{d}\theta}. \tag{9}$$

Note that the problem is assumed to be independent of the azimuth angle for simplification. In general, Z_a is also angle-dependent, except if local reaction is assumed. α_{rand} is bounded between 0 and 1.

In geometrical acoustics, $P_{inc,\infty}$ from Eq. (7) is used instead of P_{inc} from Eq. (6). With this definition of incident power, a statistical absorption coefficient is obtained,

$$\alpha_{stat} = \int_0^{\frac{\pi}{2}} \frac{8\mathrm{Re}(Z_a)}{|Z_a + Z_r|^2} \sin(\theta)\mathrm{d}\theta. \tag{10}$$

α_{stat} corresponds to the coefficient measured in a reverberation chamber and can reach values higher than 1. However, it does not always match with measured absorption data. α_{rand} and α_{stat} are related by a factor depending on Z_r, as pointed out by Thomasson [3],

$$\alpha_{rand} = \frac{\alpha_{stat}}{2\int_0^{\frac{\pi}{2}} \frac{1}{\mathrm{Re}(Z_r)} \sin(\theta)\mathrm{d}\theta}. \tag{11}$$

2.2 Case of maximum absorption

The maximum absorption scenario occurs when sound can travel freely through the absorber. In that regard, this case corresponds to an "open window" situation, in which the absorber is replaced by an aperture of the same dimensions and through which sound radiates behind the baffle. If we assume that the air properties are the same behind the baffle, this scenario corresponds to replacing Z_a by Z_r in the analogous circuit of Figure 1 in order to represent sound radiation behind the baffle.

Using Eq. (4), the reflection factor becomes

$$R(\theta) = \mathrm{j}\frac{\mathrm{Im}(Z_r)}{Z_r} \tag{12}$$

and the angle-dependent absorption coefficient from Eq. (5) is then

$$\alpha(\theta) = \frac{\mathrm{Re}(Z_r)^2}{|Z_r|^2}. \tag{13}$$

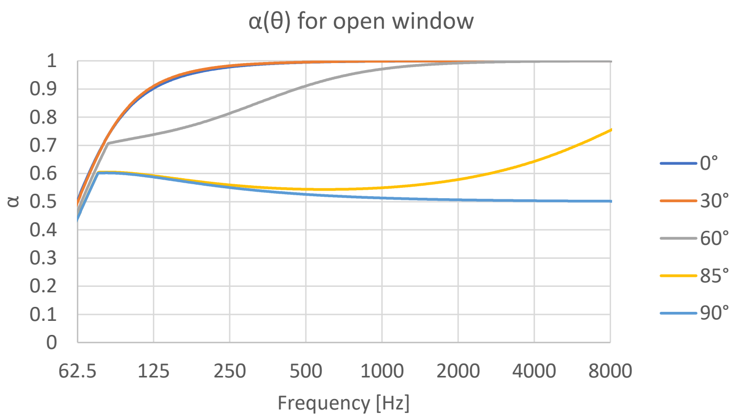

Figure 2. Angle-dependent absorption coefficient of an open window of dimensions 2 m by 2 m.

For illustration, Figure 2 shows the absorption coefficient of an open window of dimensions 2 m by 2 m as a function of frequency, for different angles of incidence. For small angles (0°, 30°), the absorption coefficient increases with frequency, until it reaches 1. This illustrates that at high frequency and close to normal incidence, the open window is totally transparent, and there are no prominent edge effects. At larger angles, α does not increase as much with frequency, which shows more important contribution from the edges. At 90° incidence, α only reaches 0.5, so at high frequencies, half of the power is transmitted through the window, while the other half keeps propagating in front. ISO 354 recommends much larger surface areas (10-12 m^2) [1], but even with such larger sizes, the absorption coefficient of the open window is below 1 for large angles of incidence, and the 90° case still converges towards 0.5 at high frequencies. This will influence sound absorption measurements, as they are obtained under a diffuse sound field, in which all angles are represented.

The aperture case was already proposed as a reference for absorption by Sabine [6]. This section illustrates that this reference definitely does not correspond to full absorption. The residual absorption is due to wave effects, which are prominent at low frequencies and at grazing incidence, as shown in Figure 2. Edge diffraction corresponds to reflection at the edges. It leads to more power being reflected, which reduces the absorption coefficient. In addition, a refraction effect occurs, leading to the bending of the incident wave towards the aperture and an increase of the absorption coefficient. This is why at 90° incidence, the absorption coefficient is not equal to 0.

2.3 Revised absorption coefficient

The proposed revised theory of absorption is based on the statement that sound absorption cannot exceed that of an aperture of the same size. Following this statement, traditional absorption coefficients are scaled by a factor $M(\theta)$, corresponding to the ratio of sound power through the window to the incident power, i.e. the absorption coefficient derived in Eq. (13),

$$M(\theta) = \frac{P_{window}}{P_{inc}} = \frac{\mathrm{Re}(Z_r)^2}{|Z_r|^2}. \tag{14}$$

The scaling corresponds to including reflection from the surroundings of the sample, which are missing in the analogous circuit approach.

We then define the following absorption coefficients using the scaling factor $M(\theta)$, in particular an angle-dependent absorption coefficient, based on Eq. (5),

$$\alpha_{revised}(\theta) = M(\theta) \cdot \alpha(\theta), \tag{15}$$

and a statistical absorption coefficient, based on Eq. (10),

$$\alpha_{stat,revised} = \int_0^{\frac{\pi}{2}} \frac{8\mathrm{Re}(Z_a)}{|Z_a + Z_r|^2} M(\theta)\sin(\theta)\mathrm{d}\theta. \tag{16}$$

3 EXAMPLES

3.1 Numerical example

We consider an absorber composed of a membrane, a porous layer and a cavity on a rigid backing. We assume the sample's dimensions are 2 m by 2 m. The configuration is modeled with the transfer matrix method in order to derive its surface impedance, using ODEON's material calculator [7]. The material is characterized by an absorption peak at about 200 Hz.

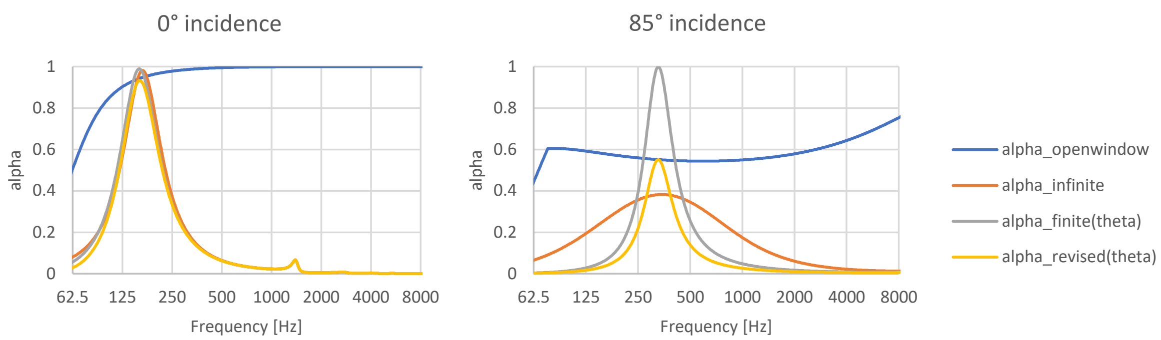

Figure 3. Angle-dependent absorption coefficients at 0° and 85° incidence. Comparison of the open window configuration Eq. (13) (blue), the infinite sample case Eq. (5) with Z_r from Eq. (3) (red), the original absorption coefficient for a finite sample Eq. (5) (gray) and the revised absorption coefficient Eq. (15) (yellow).

Figure 3 compares the original and the revised angle-dependent absorption coefficients, from Eq. (5) and Eq. (15) respectively, at 0° and 85°, together with the open window absorption coefficient (Eq. (13)) and the absorption coefficient for an infinite sample of the same material (using Z_r from Eq. (3)). At 0°, the infinite case and the traditional absorption coefficient are very similar on the whole frequency range. The open window curve shows that the revised theory will lead to a small reduction in the absorption coefficient at lower frequencies, below 500 Hz. This is visible in the revised curve, especially at the peak location. The maximum value is 0.93, compared to 0.99 for the traditional α. More drastic differences are observed at 85°, where the traditional α is much higher than the infinite case at the absorption peak. Indeed, a high α requires $Z_a \approx Z_r^*$, but this is not possible for an infinite sample, for which Z_r is purely real while Z_a has a non-zero imaginary part. The open window case approaches a value of about 0.5, as discussed in Section 2.3, which then reduces the absorption coefficient in the revised theory. According to Eq. (15), both the open window case and the traditional α constitute upper limits to the revised absorption coefficient, as both the original $\alpha(\theta)$ and $M(\theta)$ are smaller than 1.

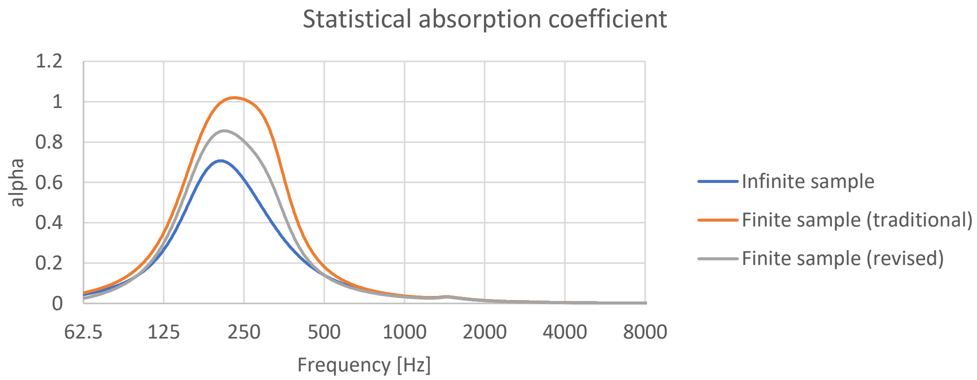

Figure 4. Statistical absorption coefficient. Comparison of the infinite sample calculation (blue), the traditional coefficient Eq. (10) (red) and the revised coefficient Eq. (16) (gray).

Figure 4 compares the traditional and the revised statistical absorption coefficients under diffuse incidence, from Eq. (10) and Eq. (16) respectively, plotted together with the infinite sample case. The revised absorption coefficient is found between α for the infinite sample and the traditional α. The revised theory tends to reduce the absorption coefficient, especially at the peak location.

3.2 Experimental data

The revised theory is also tested with experimental data measured in a reverberation chamber, and taken from Riionheimo et al. [8]. The studied configuration comprises of two 100 mm layers of porous material over a rigid backing. The first layer on the backing is made of mineral wool (mass per unit area 30 kg/m^2) and the second layer on top is polyester fiber wool (mass per unit area 20 kg/m^2). The dimensions of the sample are 3.9 m by 2.6 m.

The surface impedance and the radiation impedance of the configuration are derived with ODEON's material calculator, using a flow resistivity of 20 kN.s.m^{-4} for the mineral wool and 1750 N.s.m^{-4} for the polyester wool. From these data, the statistical absorption coefficient, the random incidence absorption coefficient, and the revised statistical absorption coefficient are derived, with Eqs. (10), (9) and (16), respectively.

Figure 5 shows these three calculated absorption coefficients together with the measured data from [8]. The data is averaged in third-octave bands. The measured absorption coefficient is the closest to the revised coefficient. As observed in Sec. 3.1, the revised statistical absorption coefficient is again situated between the traditional statistical coefficient and the random incidence absorption coefficient. In this example, the revised absorption coefficient and the measured data reach values above 1, although not as high as the traditional statistical coefficient. This case is a good example of the revised theory, because the absorption coefficient remains high down to 100 Hz, so large differences are visible between the three calculated absorption coefficients.

4 DISCUSSION

The radiation impedance is an essential tool to include finiteness effect, and it has been used in several studies already [3, 4]. In the present study, the radiation impedance is used not only to express the available power at the absorber, like in [3], but it is also included in the incident power, through Eq. (6). The main novelty is to compare the reflection process to that of an ideal open window case. The revised model thus includes reflection from the edges, which was not present in the original model. One consequence is that the revised absorption

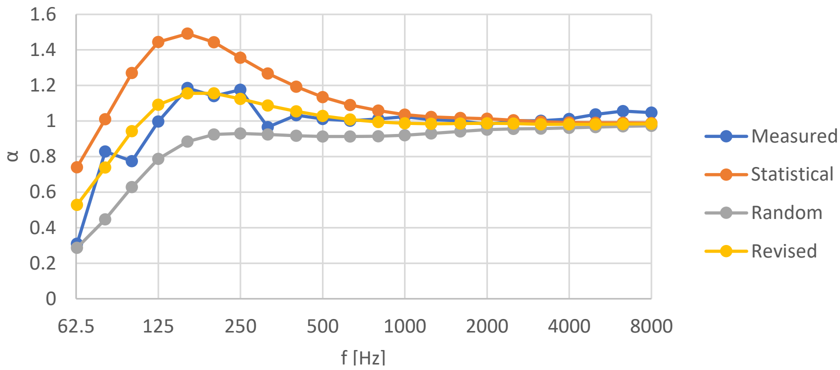

Figure 5. Measured and calculation absorption coefficients in a reverberation chamber (diffuse field). Rigid backing with 200 mm layer of polyester wool and mineral wool [8]. Statistical Eq. (10), Random incidence Eq. (9) and Revised Eq. (16).

coefficient is lower than the original one. The difference is mostly visible at low frequencies and at grazing incidence, where edge effects are more prominent.

It is known that a statistical absorption coefficient is insufficient input data to ensure fully accurate room acoustic predictions. Common issues are the lack of phase information (α being an energy quantity), the lack of angle-dependent data (it is determined at diffuse incidence) and the general lack of precision of the reverberation chamber method. The proposed revised theory does not address the phase problem as it is based on power considerations. Nevertheless, it still contributes to a better understanding of the reflection process over a finite surface. Therefore, it can still be used to predict better input absorption data for numerical acoustic models.

The comparison with experimental data showed that the revised theory can yield a better estimate of measured absorption coefficients than the traditional statistical absorption coefficient. However, the comparison can be challenging, because the measurement uncertainty can be larger than the difference between the calculated coefficients.

5 CONCLUSION

A revised theory was proposed in order to better represent reflection of sound on finite-sized absorbers. The proposed theory is based on an analogous circuit representation and assumes that absorption cannot exceed that of an aperture of the same size. The idea of a reference aperture can be traced back to Sabine's work on absorption. This reference is however not fully absorptive. Therefore, the revised absorption coefficient is lower than the traditional one, especially at low frequencies and grazing incidence. This difference is attributed to additional reflections from the surroundings of the absorber.

According to initial comparison with experimental data, the revised statistical absorption coefficient is a good candidate for predicting the measured absorption coefficient in a reverberation chamber (ISO 354). However, further investigation is recommended in order to generalize the validity of the proposed coefficient. If the agreement with reverberation chamber measurements is confirmed, the revised coefficient can be used to obtain reliable input material data for numerical prediction models, such as geometrical room acoustics.

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