The acoustics of the Pi-shaped Greek theatre in Kalydon, Aitolia

Jens Holger Rindel
Odeon AS, Scion-DTU, Building 341, DK-2800 Kgs. Lyngby, Denmark.
Rune Frederiksen
Ny Carlsberg Glyptotek, Dantes Plads 7, DK-1556 Copenhagen, Denmark.
Olympia Vikatou
Eforate of Classical Antiquities, 4, Ag. Athanasiou, 302 00 Mesolonghi, Greece.

Summary
The ancient Greek theatre in Kalydon (αρχαία Καλυδώνα), Aitolia, has a special shape with a square orchestra and a Pi-shaped koilon. The seat-rows of the lower section (rows 1 to 9) are straight and meet at a right angle between the wings and the central part, whereas the seat-rows of the upper section (rows 10 to 31) are connected by curved seat-rows in the shape of a quarter-circle. The Skene building had ramps to both sides and the height of the proskenion has been approximately 2.5 m above the orchestra. The theatre dates from the late 4th or early 3rd century BC. The acoustical properties of the theatre are analysed in a computer model of a reconstruction of the theatre. Two sound sources are used; an omnidirectional source and a source with the directivity and spectrum of a speaking person. The acoustical parameters used for the analysis are the sound pressure level, the clarity index $C_{50}$, the speech transmission index STI and the Dietsch echo criterion. Another analysis is made on a theatre model with semi-circular koilon, but using the same slope of the seat-rows and the same skene building. This theatre has a circular orchestra with the same area as the square orchestra in the Kalydon theatre (ca. 245 m$^2$). By comparison of the acoustical results from the two models it is discussed whether or not the acoustical properties have been a major reason for the evolution of the ancient Greek theatres from the Pi-shape to the semi-circular shape.

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1. Introduction
The typical semi-circular shape of the koilon (cavia, audience area) in Greek theatres was probably developed in the second half of the 4th century BC [1, p. 68; 2]. Earlier theatres had rectilinear seat rows in various shapes, like the rhombic shape of the first theatre in Syracuse (Sicily) that dates before 409 BC, the theatre at Trakhones, Attica, and the theatre at Argos, which had at least 33 seat-rows [2]. The second phase of the theatre at Thorikos, Attica, dates to between 480 and 425 BC and this had 21 seat-rows running straight for some 20 m and bend at both sides [2].

The theatre in Kalydon, Aitolia, is particularly interesting, because the archaeological excavation has provided sufficient information for a reconstruction, and the design of the theatre is strictly geometrical [3]. It dates from the 4th century BC. Kalydon was a prosperous city until the end of the 1st century BC. Emperor Augustus removed the population of Kalydon to the new colony of Nicopolis in 31 BC, and the city was abandoned [4]. This may be the reason that the theatre has not been changed, unlike many other Greek theatres.

The remains of the theatre were discovered in 1963, and in 2002, a Danish-Greek project resumed systematic investigations at Kalydon. The excavation of the theatre was made 2011-2014 [3] and a monography is under preparation. The aerial view from the site in Figure 1 shows the three parts of the theatre: the square orchestra, a part of the koilon and the foundation of the scene-building. The koilon is facing South.
2. Description of the theatre

2.1. Orchestra

The central part of the theatre is the orchestra, which is nearly square, about 15.6 m wide and 14.8 m deep if measured from the first seat row to the columns in the proskenion. The surface is beaten earth, i.e. hard and highly reflective to sound. The orchestra has a slight slope of 0.20 m from the NW corner to the SE corner, which might have had the purpose of leading away rainwater.

2.2. Koilon

The koilon is shaped as the letter Π (Pi), i.e. the first nine seat rows form three sides of a rectangle with 90° between the centre section and the two side sections. The rows from number 10 up to at least 30 have straight sections that meet in rounded corners, shaped as quarter-circles. The analemma (wall supporting the koilon on both sides of the orchestra) is parallel with the seat-rows in the centre section and parallel to the proskenion. Unlike most later theatres, there is no subdivision of the koilon, neither by a horizontal diazoma, nor by stairways. The height of the seat steps is 0.30 m, whereas the depth varies. The first five steps are about 0.60 m deep; the next three steps are 0.75 m, 0.85 m and 0.80 m, respectively. The following steps from row nine are around 0.77 m deep. The average seat depth of the 31 rows is 0.745 m. The total seating area can be calculated to around 1900 m² corresponding to a maximum capacity around 5000 people.

2.3. Skene building

As a result of the archaeological excavation it is possible to describe the skene building. The proskenion has a completely preserved stylobate, which held a row of 10 pillars in Ionic order. The height of the proskenion can be estimated to about 2.5 m. The width of the skene building was about 13 m, and sloping ramps on both sides provided access to the proskenion (or logeion, pulpitum, stage). The scaenae frons (façade of the skene building) was about 2.4 m behind the proskenion, so this was the depth of the logeion. The height of the skene building is estimated to about 5.85 m plus the roof. The capitals of the proskenion pillars have indicated a construction date in the 4th century BC.

The sandstones used for the skene building are similar to those used for the koilon.

3. Semi-circular theatre for comparison

For the evaluation of the acoustics of the Π-shaped theatre in Kalydon, it is interesting to make a comparison with a semi-circular theatre with equivalent dimensions. Thus, two computer models were created for the acoustical simulations, one as the reconstructed Kalydon theatre and the other one with semi-circular shape. The skene building, the centre of the orchestra and the slope of the seat-rows are the same in both models. The centre of the orchestra is defined as the crossing point of the 45° diagonals in the Π-shaped theatre. The diameter of the circular orchestra (17.6 m) was chosen so that the area equals that of a square with the side 15.6 m. Some semi-circular theatres with a similar design have been identified, see Table I.

<table>
<thead>
<tr>
<th>Place</th>
<th>( D ) (m)</th>
<th>Seats (cm)</th>
<th>( \alpha ) (°)</th>
<th>Number of rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morgantina</td>
<td>14.4</td>
<td>34 × 70</td>
<td>25.9</td>
<td>15+</td>
</tr>
<tr>
<td>Segesta</td>
<td>14.8</td>
<td>38 × 73.5</td>
<td>27.3</td>
<td>21+</td>
</tr>
<tr>
<td>Soluntum</td>
<td>14</td>
<td>38 × 71</td>
<td>28.2</td>
<td>23+</td>
</tr>
<tr>
<td>Iaitas (model)</td>
<td>21.5</td>
<td>39 × 74</td>
<td>27.8</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>17.6</td>
<td>30 × 74.5</td>
<td>21.9</td>
<td>31</td>
</tr>
</tbody>
</table>

The theatre of Stratos, Aitolia, is located quite close to Kalydon, and this theatre has many similarities with the Kalydon theatre, but with a circular orchestra and a semi-circular koilon. It dates
probably from the late 4th century BC. Unfortunately, the excavation of this theatre has not yet been reported and the necessary data for creating a model are not available. It appears from Table I that the other identified semi-circular theatres have slightly steeper slope of the seats-rows than the Kalydon theatre (around 27° and 22°, respectively). The plan of the reconstructed Kalydon theatre is seen in Figure 2, and that of the semi-circular theatre is seen in Figure 3.

![Figure 2. Plan of model of the Kalydon theatre. Maximum audience is around 5000.](image)

![Figure 3. Plan of model of the semi-circular theatre. Maximum audience is around 4300.](image)

Although the number of seat-rows is the same in both theatre models (31) and the orchestra has the same area, the seating area in the semi-circular theatre is only 1600 m² corresponding to a maximum capacity around 4300 people. Compared to the semi-circular theatre, the Π-shaped theatre has some seats closer to the centre of the orchestra, but also many seats at a longer distance. The koilon of the semi-circular theatre is extended above the 180° by a small straight section, so the analemmata are in the same distance from the skene in both theatres. Actually, this means that the shape of the orchestra is strictly speaking more like a horse-shoe. This shape of the koilon is very similar in the theatres of Segesta and Iaitas [1].

4. Acoustical modelling

4.1. Sources

Two different sound sources are applied in the acoustical calculations. One source is the ideal point source with omni-directional radiation, as this represents the sound source usually applied for acoustical measurements in auditoria, see ISO 3382-1 [5]. Equal sound power is assumed in all octave bands.

The other sound source has the directivity and frequency spectrum of speech. The vocal effort is set to ‘loud’ in accordance with ISO 9921 [6], which means that the A-weighted sound pressure level in a distance of 1 m in front of the mouth is 72 dB. The height of the source is in all cases 1.5 m above the floor/ground.

4.2. Materials

The acoustical properties of the materials are very simple, either very reflective (stone, beaten earth) or very absorptive (the audience), see Table II.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Hard surfaces</th>
<th>Audience</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>0.02</td>
<td>0.16</td>
</tr>
<tr>
<td>250</td>
<td>0.03</td>
<td>0.24</td>
</tr>
<tr>
<td>500</td>
<td>0.03</td>
<td>0.56</td>
</tr>
<tr>
<td>1000</td>
<td>0.03</td>
<td>0.69</td>
</tr>
<tr>
<td>2000</td>
<td>0.04</td>
<td>0.81</td>
</tr>
<tr>
<td>4000</td>
<td>0.07</td>
<td>0.78</td>
</tr>
</tbody>
</table>
4.3. Early reflections

With the source on the proskenion, the sound propagation to a receiver sitting on row 28 is seen in Figure 4, upper part. The direct sound is followed by two first order reflections with delay times of 6 ms and one second order reflection with 12 ms delay. The reflecting surfaces are the orchestra and the scenaes frons of the skene building. These reflections are very beneficial as they increase the sound level and the clarity.

The lower part of Figure 4 shows the situation with a sound source in the centre of the orchestra. The direct sound is followed by a reflection from the orchestra with 3 ms delay. The reflection path follows the slope of the seats, and thus it will be attenuated in case of a full audience. However, the two reflections from the skene arrive with nearly 60 ms delay. Since the delays exceed 50 ms, these reflections are heard as a distinct echo, and consequently the speech intelligibility is reduced.

Figure 4. Section with early reflections to a receiver on row 28. Upper: Source on proskenion. Lower: Source in centre of orchestra. Red dots indicate the image sources.

The analysis in Figure 4 supports the assumption that the main actors with speech roles have performed on the proskenion, not on the orchestra. While the sight-lines to the orchestra are excellent, the sound from sources in the orchestra was not good. Only the part of the orchestra less than 8.5 m from the skene building could be used without the echo problem, e.g. for the chorus or music.

Figure 5 shows early reflections in the same case as in Figure 4, upper part. Reflections are possible from the side wings of the koilon, and having a delay of about 65 ms, these reflections may cause echo problems. With a full audience, the reflections are attenuated due to the sound absorption of people, so the echo may not be a serious issue. Similar delayed reflections from one part of the koilon to another are possible in other parts of the theatre, see section 5.4.

Figure 5. 3-D view of the model with propagation paths of early reflections from the source on proskenion. Echo may occur due to lateral reflections from the koilon.

5. Results

5.1. General

The acoustical calculations were made with the software ODEON, version 14.03 [7]. Grid maps of the results were calculated using a grid density of 0.75 m and a height of 0.80 m above the seats. Number of rays were set to 100 000 and the transition order was 2. The calculated impulse responses were 2.0 s. The scattering coefficient at mid-frequencies was set to 0.05 for all surfaces.

5.2. Sound pressure level

A sound pressure level of speech above 40 dB it is good for an outdoor situation, if the background noise is below 30 dB. Figure 6 shows a map of the calculated sound pressure level in the Kalydon theatre. The average is 46 dB and 90 % is within the range 41 dB to 51 dB. Of course, the best seats are close to the source, whereas the weak areas are in the corners more than 30 m from the source.

Figure 7 shows the sound pressure level in the semi-circular theatre. The weak areas are smaller and moved to the sides of the koilon. The average is 46 dB and 90 % is within the range 43 dB to 51 dB. The average is the same, but there is an improvement of the sound level in the weakest areas in the theatre. Figure 7 also shows that the decrease of sound level with distance follows concentric circles that coincide with the seat-rows.
5.3. Clarity of speech

The balance between early and late arriving sound energy is the clarity in dB and it is defined in ISO 3382-1 [5]. For speech the early time limit is 50 ms and the symbol is $C_{50}$. Values above 10 dB are very good. The clarity was calculated using the omni-directional sound source.

Figure 8 shows a map of the calculated speech clarity in the Kalydon theatre. The average is 14 dB and 90 % is within the range 11 dB to 20 dB. The highest clarity is in the remote seats to the left and right of the orchestra. However, this does not mean that these seats are particularly good; it only indicates that little energy arrives after 50 ms. From Figure 6 we know that the sound pressure level is relatively weak in these areas.

Figure 9 shows the speech clarity in the semi-circular theatre. The best area is in the centre part of the koilon, where the improvement is striking. The average is 16 dB and 90 % is within the range 11 dB to 21 dB. The improvement is 2 dB on average, and more (about 5 dB) in the middle of the koilon.

5.4. Speech Transmission Index

The Speech Transmission Index (STI) is a measure of the quality of vocal communication. Like $C_{50}$ it takes into account the balance between early and late arriving sound, but in addition the background noise and the speech level and frequency spectrum are applied. For the calculations was used a speech source as described in section 4.1. The A-weighted sound pressure level of the background noise was set to 25 dB with equal level in all octave bands (pink noise). STI can take values between 0 and 1. STI > 0.75 is excellent, and values between 0.60 and 0.75 are good [6].

Figure 10 shows a map of the calculated STI in the Kalydon theatre. The average is 0.80 and 90 % is within the range 0.71 to 0.88. In general, this is very good. The weakest areas are in the remote corners of the koilon, the same areas that have low sound pressure level and clarity, see Figures 6 and 8.

Figure 11 shows the STI in the semi-circular theatre. The result is improved in most of the koilon. The average is 0.83 and 90 % is within the range 0.74 to 0.90.
Figure 10. Distribution of speech transmission index STI in Π-shaped theatre.

Figure 11. Distribution of speech transmission index STI in semi-circular theatre.

5.5. Echo criterion

The echo criterion for speech signals by Dietsch & Kragh [8] was calculated using the omni-directional source. The parameter is usually between 0 and 1, but it can take values > 1. Low values are good. It is estimated that more than 90% are annoyed by echo if the parameter exceeds 1.5, and 50% are annoyed by when the parameter is 1.

Figure 12 shows a map of the calculated echo parameter in the Kalydon theatre. The average is 0.56 and 90% is within the range 0.31 to 0.94. In the remote corners of the koilon there are small spots where the value exceeds 1.

Figure 13 shows the echo parameter in the semi-circular theatre. The average is 0.56 and 90% is within the range 0.30 to 1.00. The highest risk of echo is in the extreme sides of the theatre, coinciding with the areas with the lowest STI (compare with Figure 11). The areas with improved conditions coincide with those with improved clarity $C_{50}$ (compare with Figure 9).

6. Discussion

The plan of the Kalydon theatre leaves no doubt that it has been designed with great care using geometrical shapes, the square, the diagonals and the quarter circles. In the 4th century BC, geometry had been developed to a very high level in Greece. Known scientists of the time include the mathematician Eudoxus (c 391 – 338 BC) and Aristoxenus (c 375 – 335 BC). The latter is known for his treatises on sound and music among many other topics. We know that Aristoxenus was particularly interested in the acoustics of theatres, because he is the origin of the unfortunate idea to install sounding vessels in theatres as mentioned by Vitruvius [9, Book V, Chapter V].

The acoustical analysis of the Kalydon theatre shows that the sound was excellent in the majority of the audience area, however with less satisfactory acoustics in the remote corner sections. With a source position on the proskenion, the sound received in the middle part of the theatre was supported by three sound reflections with very short delay times.
The orchestra was the most important reflecting surface in the theatre. Figure 14 shows the coverage of the first order reflections from the orchestra on the audience area in the Kalydon theatre. For comparison, Figure 15 shows the coverage of the first order reflections from the orchestra on the audience area in the semi-circular theatre. Although the area of the orchestra is approximately the same, the reflections from the orchestra cover a larger part of the audience in the semi-circular theatre. The orchestra is more efficient as a sound reflector. The transition from the Π-shape to the semi-circle improved the acoustics of the theatres from very good to better. The only drawback may be the reduced seating capacity with the same number of seat rows. Since the semi-circular theatre became the canonical shape, we can say that quality was preferred over quantity.

There is little doubt that the ancient Greek designers of the theatre were aware of the importance of the orchestra as a sound reflector. Vitruvius writing on the design of theatres is partly a translation of ancient Greek treatises on acoustics, and he explains four different kinds of sound reflections with their Greek names [9, Book V, Chapter VIII]. Some are destructive like the _resonant_ reflection, which can produce an echo. Most beneficial are the _consonant_ reflections, in which the voice ‘is supported from below, increases as it goes up, and reaches the ears in words which are distinct and clear in tone’ (Citation from [9]).

A further analysis can be made when looking at the vertical section through the centre axis of the theatre, Figure 16. This analysis is inspired by the work of Canac [10], who demonstrated the importance of the sound reflection from the orchestra in antique Hellenistic and Roman theatres.

In Figure 16, $D$ is the horizontal distance from the source to the first seat row, i.e. approximately the depth of the orchestra plus a small correction. This is the distance from the _proskenion_ to the middle of the _proskenion_, roughly about 1 m.

The height of the source $S$ above the plane of the orchestra is $h$, and path of the sound reflection follows the line from the image source $S'$, which is below the plane of the orchestra.

The slope of the seat rows is characterised by the angle $\alpha$, where $\tan(\alpha)$ is the height of the steps divided by the depth. The sound reflection from the orchestra arrives to the receiver $R$ on the last row from a direction, which is determined by the line from the image source $S'$.

The angle $\epsilon$ indicate the direction of the reflected sound relative to the slope of the seat rows $\alpha$. This angle is crucial, because the sound is attenuated if this angle is close to zero. If $\epsilon$ is negative the sound reflection has disappeared. With reference to symbols in Figure 16, $\epsilon$ is calculated from the equation:

$$\tan(\alpha - \epsilon) = \frac{H+h}{D+L} \quad (1)$$
Inserting the dimensions for the last row 31 of the Kalydon theatre ($\alpha = 21.9^\circ$, $H = 10.0$ m, $h = 4.0$ m, $L = 23.1$ m, $D = 15.6$ m) yields $\varepsilon = 1.7^\circ$. Moving down to row 11, $\varepsilon$ increases to 2.8°. The last row is always the most critical.

Equation (1) may also be used as a key for designing the proportions of a theatre. If the slope $\alpha$ of the kolinon is fixed, the number of rows necessary to obtain the desired maximum capacity determines $H$ and $L$. The height of the proskenion plus 1.5 m (height of mouth of a standing person) defines $h$. Then, with $\varepsilon = 2^\circ$, the minimum dimension $D$ of the orchestra can be calculated from Equation (1). This also explains why a smaller orchestra may be sufficient if the height of the proskenion is reduced, as in the case of a Roman theatre.

The geometrical design principle with the orchestra as the main acoustic reflector is probably the key to the good acoustics in the Kalydon theatre. As suggested by Canac [10], this principle may have been applied for the design of antique theatres from the Hellenistic and Roman periods.

After the decline of the Roman empire, the theatres were no longer being used, and the knowledge was lost. The information conveyed by Vitruvius was not sufficient for understanding the acoustical principles. Instead, confusion and misunderstandings arose from his description of the mysterious sounding vessels, an unfortunate mistake by Aristoxenus that has survive to this day.

### 7. Conclusion

The acoustics has been analysed in a reconstruction of the Π-shaped theatre in Kalydon. With a sound source standing on the proskenion and speaking with loud voice, the sound pressure level varies within 41 dB and 51 dB (90% within these limits), the clarity $C_{50}$ varies within 11 dB and 20 dB, and the Speech Transmission Index STI varies within 0.71 and 0.88. Thus, the acoustics may be characterised as very good for speech.

The main reason for the good acoustics is efficient sound reflection from the orchestra. This reflection covers a major part of the audience area and increases the sound level and clarity.

Comparison of the results with those from a similar analysis of a semi-circular theatre has shown that all acoustical parameters improve. Especially the middle part of the audience area has a remarkable improvement of the clarity of speech.

The orchestra as a sound reflector is more efficient in the semi-circular theatre than in the Π-shaped theatre. Although the area of the orchestra is approximately the same in the two theatres, the audience area covered with reflections from the orchestra is larger in the semi-circular theatre. However, the seating capacity is smaller in the semi-circular theatre (about 85%).

Canac [10] has previously suggested that a thorough geometrical design principle was generally applied in antique theatres for obtaining excellent acoustics. The present work supports this hypothesis, and the Kalydon theatre may be one of the first and oldest examples of thorough geometrical design for good acoustics.

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### References